MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY

CONTAINING

PAPERS, ABSTRACTS OF PAPERS, AND REPORTS
OF THE PROCEEDINGS OF THE SOCIETY
1943

VOL. 103

Published and Sold by the
ROYAL ASTRONOMICAL SOCIETY
BURLINGTON HOUSE
LONDON, W. 1
1943

Price £1, 16*s.*

Printed in Great Britain by
Neill & Co., Ltd., Edinburgh

MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY

Vol. 103 No. 1

The Council elected the following as Fellows of the Society at the Council meeting on 1943 February 12:—

Kenneth Charles Blackwell, Royal Observatory, Greenwich, London, S.E. 10;
Frederick Richard Cripps, 6 Belmont Road, Bushey, Watford, Hertfordshire;
Colin Alistair Ronan, 6 College Road, Ealing, London, W. 13; and
Cyril Arnold Williams, Monkton House School, The Parade, Cardiff, South Wales.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Herbert Coates, M.A., Mill Hill School, St. Bees, Cumberland (proposed by David Brunt);
Albert William Francis Hallam, 44 Parklands Road, Streatham Park, London, S.W. 16 (proposed by F. M. Holborn);
William S. Mason, 62 Grafton Road, Auckland, New Zealand (proposed by R. A. McIntosh);
Douglas Walter Noble Stibbs, Lecturer in Physics and Mathematics, New England University College, Armidale, New South Wales, Australia (proposed by R. v. d. R. Woolley);
Oliver Rowland Walkey, Stanford Bishop Vicarage, Bringsty, Worcester (proposed by Arthur R. Hinks); and
John Douglas Ford Williams, B.Sc., Lecturer in Physics, Coventry Technical College, Coventry (proposed by J. P. M. Prentice).

MEETING OF 1943 MARCH 12

Professor S. Chapman, President, in the Chair

The election by the Council of the following Fellows were duly confirmed:—

John Thornton Burge, Glenhurst, Braeside Road, West Moors, Dorset;
Ben Burrell, 93 Woodhouse Road, Doncaster, Yorkshire;
Henry J. Green, M.Inst.C.E., M.I.Mech.E., Corvette, Church Lane, Pinner, Mddx.;
Aylmer Joseph Probyn Maude, 3 Airlie Gardens, London, W. 8;
Frank Peverett, 7 Grandage Terrace, Whetley Hill, Bradford, Yorkshire;
John Benson Sidgwick, Greenfields, Llanfair Road, Newtown, Mid Wales;
Henry Bertram Tetlow, 2 Lymm Hall, Lymm, Cheshire; and
Vladimir Vand, D.Sc., 32 Temple Fortune Hill, London, N.W. 11.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

William Mattock Mason, Airton House, Airton, near Skipton, Yorkshire (proposed by William Porthouse);

Richard Griffith Quirk, A.I.N.A., 23 Eden Quay, Dublin (proposed by H. Freyne); and

David C. West, B.Sc., 125 Sherbrooke Street W., Montreal, Canada (proposed by A. Vibert Douglas).

Eighty-two presents were announced as having been received since the last meeting, including—

Abastuman, Astrophysical Observatory of the Academy of Sciences of the Georgian S.S.R., Bulletin, No. 6 (presented by the Astrophysical Observatory, Abastuman);

Association of Special Libraries and Information Bureaux, War-time guides to British sources of specialised information. No. 4. Scientific and technical periodicals received from the U.S.S.R. (presented by the Association of Special Libraries and Information Bureaux);

British Association for the Advancement of Science, Seismological Tables by Harold Jeffreys and K. E. Bullen (presented by the Gray Milne Trust);

Monthly Astronomical News-Letter, Nos. 4, 5 (presented by the Committee for the Distribution of Astronomical Literature);

H. Alan Lloyd, *The work of Samuel Watson of Coventry and London (*ca.* 1635 to *ca.* 1710) (presented by the author);

C. E. Kenneth Mees, The Theory of the Photographic Process (presented by the author);

New York, Federal Works Agency, Works Projects Administration Tables, Vols. 11, 13, 14 (presented by National Bureau of Standards);

G. Shajn, *Results of observations of the corona spectrum, June 16, 1936 at Omsk (presented by the author);

G. Shajn, *The Occurrence of carbon isotopes in the spectra of N type stars (presented by the author);

F. Twyman, Prism and Lens Making. A Text Book for optical glass workers (presented by the author);

R. L. Waterfield, The Revolving Heavens (presented by the author);

Books and slides belonging to the late W. G. Bell (presented by Mrs. Bell); and

A Celestial Globe and 6-inch Object Glass used by the late W. F. Denning (presented by his sister, Mrs. Willett).

An asterisk (*) *indicates that the work is an excerpt.*

The President announced that the Council had awarded the Gold Medal of the Society to Dr. H. Spencer Jones, M.A., Sc.D., F.R.S., Astronomer Royal, for his determination of the solar parallax.

The President also announced that Dr. A. D. Thackeray had resigned from the Secretaryship and that the Council had co-opted Dr. R. H. Hulme on to the Council in his place.

The President further announced that at the Ordinary Meeting of the Society in May an address would be given by Professor H. Dingle on Copernicus to commemorate the 400th anniversary of his death.

LINE-WIDTH MEASURES AND THE OBSERVATION OF A RED-SHIFT IN THE ERUPTIVE *Hα* LINE

M. A. Ellison

(Received 1943 January 11)

Summary

The evidence is reviewed for the existence of vertical and horizontal movements in the bright *Hα* emission of chromospheric eruptions.

187 measures of line-width, made upon 22 eruptions, show a red-shift increasing with line-width and visual intensity. The value is about +0·3 A. for the brightest eruptions. The measures have been made in all cases upon the wings of the emission line and are relative to the centre of the *Hα* absorption line for the surrounding field. The data do not appear to favour a Doppler interpretation. The suggestion is made that the observed red-shift arises from increased absorption on the violet side of the emission line, which may have its origin in the general expulsion at high velocity of absorbing matter from the eruption as centre.

Line-width measures have been plotted, as a function of time, during the progress of 10 eruptions. These curves indicate the same general sequence of events for all eruptions—a rapid rise to maximum brightness and line-width followed by a slow decline, the durations of rise and fall being in the ratio of about 1 to 4. In two Class III eruptions fluctuations of line-width have occurred after the peak value.

The question of horizontal movements in the bright emission is discussed, and it is concluded from observations on 95 eruptions that, while the displacement caused by a steady velocity of 1 km./sec. could be easily seen and measured in favourable cases, no such movements have been certainly observed.

---

1. *Introduction.*—It is now generally agreed * that large displacements, attributable to sightline velocities, have never been observed with certainty in the bright emission lines of chromospheric eruptions.

On the other hand, it has been shown by Newton † and Ellison‡ that the bright emission serves as a centre from which, at the time of greatest intensity, high-velocity absorption flocculi are ejected in a large percentage of cases (80 per cent. for the brightest eruptions, after Newton). The characteristic motions of these flocculi have been investigated by both observers, who have analysed the Doppler displacements in the absorption lines of many such markings. Vertical velocities in the range 30–40 km./sec. are most frequently met with, but values (in ascent) of > 100 km./sec. are not uncommon. When the eruption flocculi are observed in profile at the limb they appear as "metallic" prominences of more than average brightness, rising to heights of 30,000 km. or more before falling back—at least partially—towards the chromosphere. It seems to be well established that these ejections should be classified as *quasi-eruptive* prominences (II*a* of Pettit's classification §). Their association with a visible eruption and their eventual return to the chromosphere set them apart from the more extensive *eruptive prominences* (II of Pettit's classification). The latter represent, in all probability, the final "blow-

* See, for example, the views of Giovanelli (*Ap. J.*, 91, 340, 1940) and Waldmeier (*Zs. für Ap.*, 20, 46, 1940).

† *M.N.*, 102, 2, 1942.

‡ *M.N.*, 102, 11, 1942.

§ *Proc. Amer. Phil. Soc.*, 79, 488, 1938.

off" stage in the life of a typical quiescent dark flocculus, and the available evidence * indicates that the ascents of these prominences are not uniquely associated with visible chromospheric eruptions.

These facts have suggested that the occasional reports in the *Quarterly Bulletin on Solar Activity* of bright eruptions which have exhibited measurable sightline velocities might be attributed to a failure to distinguish between the stationary patches of bright emission and the moving prominences to which they give rise. When seen at the limb, these filaments are of such brilliance—relative to normal prominences—that they may be readily confused with the bright eruption itself, lying at a lower chromospheric level. Again, Giovanelli † has stated that bright eruptions normally show little or no vertical velocity and has, in support of his view, given displacement measures of the *Hα* line

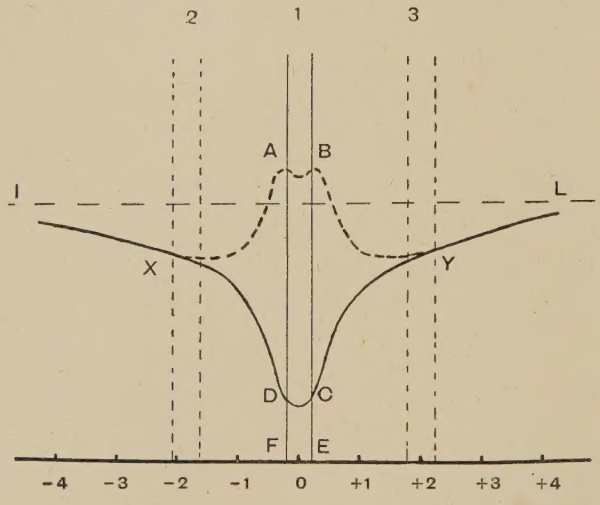


FIG. 1.—*Diagrammatic representation of Hα absorption contour* (XDCY) *with the eruption emission contour* (XABY) *superimposed. The vertical lines represent successive positions of the spectrohelioscope viewing slit* (0·45 *A. wide*). IL *is the intensity level of the continuous spectrum.*

(taking the mean of the extinction values on the wings of the emission line) for 138 eruptions. Interpreting his very small displacements as due to Doppler effect, he finds that recessive velocities outnumber those of approach by 92 to 35, with 11 of zero velocity, giving a mean for all of +3·4 km./sec. Thus any motion is, on the average, inwards along the solar radius.

In view of these uncertainties, the work of the Sherborne spectrohelioscope has been directed during the past two years to collecting all possible information which might have a bearing upon the movements, both lateral and vertical, of bright eruptions. During the course of this work a small, but easily measurable, red-shift has been noted, occurring with great regularity in the *Hα* line near the time of greatest intensity of the bright emission. Though the data relating to it are not so extensive as could be wished, it may be of value to publish the measures which have already been obtained, since it is unlikely that many more will be forthcoming before the next period of sunspot maximum. By that time, it is hoped, the problem may be tackled more effectively in a different way. Microphotometer tracings obtained from a number of spectrograms taken during the course of a single eruption would undoubtedly yield more information about the contour and displacement of the *Hα* line than can be derived visually with the spectrohelioscope. The spectrum of the bright emission has been photographed (under

* *Ap. J.*, **88**, 244, 1938.

† *Ap. J.*, **91**, 340, 1940.

high dispersion) by Allen * and others, but probably not in the early stages of a Class III eruption when the line-width and red-shift are found to reach a maximum.

2. *Methods of Measurement.*—In fig. 1, let XABY represent the contour of the *Hα* emission line in an eruption, and XDCY the normal *Hα* contour of the *surrounding background.* Placing the slit of the spectrohelioscope centrally upon the absorption line in the position ABEF, we observe the bright eruption with its maximum contrast against the mottled *Hα* of the chromosphere. We may define this contrast as $c-1$, where $c=I_f/I_d$, I_f and I_d being the intensities of flocculus and disk for the same wave-length.† When we take into account the finite slit-width (0·1 mm. = 0·45 A.) of the instrument, I_f and I_d will be represented by the areas ABEF and DCEF respectively. For bright flocculi and bright eruptions the contrast may have values lying between zero and an upper limit of about +20 which would correspond to a Class III eruption with a central intensity 315 per cent. of the continuous spectrum.

As the viewing slit of the spectrohelioscope is moved to the long- or short-wave sides of normal *Hα*, the contrast falls rapidly until its visual appreciation (proportional to log *c*) becomes zero at the points X, Y where the absorption and emission lines blend into one another. The wave-lengths corresponding to the points X, Y will be called the *extinction values* for the eruption, while the wave-length interval between the points will be referred to as the *effective width* of the emission line.

The disappearance of the eruption in the slit positions X, Y may be judged with a probable error under 0·1 A.; for the conditions are comparable with those in the best type of disappearance photometer (Lummer-Brodhun). Furthermore, the estimation is almost independent of atmospheric clarity and seeing. In practice the slit position remains fixed during an observation and the spectral image is caused to traverse it by the rotation of the line-shifter. Thus each measure of effective line-width consists of five line-shifter settings—three settings in position 1 for the centre of the absorption line (darkest field) and one each in positions 2 and 3, the slit being moved always outwards from the centre of *Hα*.

3. *Results.*—One hundred and eighty-seven measures have been made, on the average at three-minute intervals, during the progress of 22 eruptions. The eruptions were placed in the following categories: 3 of Class III, 8 of Class II, and 11 of Class I. The mean of the extinction values has been taken for each measurement of effective line-width, and, subtracting from this the wave-length of normal *Hα* (6562·8), we have obtained the displacement (+ for a red-shift and − for a blue-shift). These values have been incorporated in Table I, where the mean displacements of the emission line are shown in Column 2 for different intervals of effective line-width, together with the number of measures (Column 3) made in each of these intervals. Columns 4 and 5 contain, for comparison, 161 measures made upon 25 areas of quiescent bright hydrogen.

For the eruptions it will be seen that the means show a positive (red) shift for all values of line-width lying between 1·75 and 5 A., becoming steadily greater as the line-width increases. This is illustrated graphically in fig. 2, where the mean displacements for quiescent bright hydrogen are also plotted. It may be mentioned here that the line-width of an eruption is roughly proportional to its brightness as observed on the centre of the emission line. This conclusion is supported by the work of both Giovanelli and Waldmeier. Thus we may say that the red-shift increases with the brightness of the eruption. The same tendency seems to be manifest in the quiescent hydrogen, but here the range of measurement is limited, since areas that could be classed as quiescent have shown no line-widths greater than 2·3 A. The greatest value observed for eruptions was 6·1 A. (Class II: 1942 November 14, $11^h\ 13^m$ U.T.) and experience suggests that this value may often be exceeded for a short time (one or two minutes only) at the peak intensity of some Class III eruptions.

It may be emphasised that the red-shift is intimately connected with the growth and

* *M.N.*, 100, 635, 1940. † *The Observatory*, 64, 300, 1942.

decay of each eruption, and is not previously inherent in the bright hydrogen from which the eruption was generated. It appears as the eruption commences and increases rapidly with the increase of intensity and line-width, reaching a maximum value at the same time as the peak intensity of the light; thereafter it decreases slowly as the eruption fades away to its original brightness. The greatest value appears to be in the neighbourhood of +0·3 A. for the brightest eruptions.

It seemed at first as if the phenomenon could be most simply explained by the existence of some systematic error in the calibration of the line-shifter scale. However, the original calibration has been verified as correct by an independent method.

4. *Interpretation.*—The symmetry of the wings in the normal *Hα* absorption line has been sufficiently established, photographically by Thackeray * and visually by Woolley and Newton.† It is true that Thackeray's values indicate slightly greater

TABLE I

Displacements of the Hα Emission Line in Bright Eruptions

Effective Width of *Hα* Emission Line	Eruptive Bright Hydrogen (22 Eruptions)		Quiescent Bright Hydrogen (25 Areas)	
	Mean Displacement, Red-shift + Blue-shift −	No. of Measures	Mean Displacement, Red-shift + Blue-shift −	No. of Measures
Angstroms	Angstroms		Angstroms	
0·75–1·00			−0·047	3
1·00–1·25	−0·065	4	−0·045	16
1·25–1·50	−0·024	17	−0·033	29
1·50–1·75	−0·018	25	−0·014	49
1·75–2·00	+0·030	23	+0·008	38
2·00–2·25	+0·068	28	+0·051	21
2·25–2·50	+0·109	33	+0·050	5
2·50–2·75	+0·120	36		
2·75–3·00	+0·123	9		Total = 161
3·00–3·25	+0·148	5		
3·25–3·50	+0·250	1		
3·50–3·75	+0·200	1		
3·75–4·00				
4·00–4·25	+0·280	1		
4·25–4·50	+0·255	2		
4·50–4·75	+0·190	1		
4·75–5·00	+0·260	1		
		Total = 187		

absorption on the violet side of normal *Hα*, but in the nature of the above measurements this might be expected to cause a violet-shift in the emission line. We must therefore attribute the observed red-shift either to an asymmetry in the wings or to a genuine displacement of the emission line as a whole.

The possibility of an unsymmetrical Stark effect may be ruled out in the present instance by reason of the impossibly large fields required to produce it. If we assume that the red-shift arises from a simple Doppler displacement, it follows that the brightest eruptions are descending into the chromosphere at a speed of about 14 km./sec. and the fainter ones at a proportionally smaller rate. Thus, for a Class III eruption of long duration, like that of 1942 February 28,‡ where the red-shift was still in evidence $1^h\ 45^m$ after the commencement of the eruption, the Doppler interpretation would imply a vertical descent of at least 50,000 km. during that period. So great a vertical displace-

* *M.N.*, 95, 295, 1935. † *M.N.*, 96, 9, 1935. ‡ *The Observatory*, 64, 260, June 1942.

ment does not seem to be at all likely on theoretical grounds, nor consistent with the observations of bright eruptions occurring near the limb. Moreover, the Doppler interpretation would lead us to expect a displacement of the line varying with the cosine of the longitude from the central meridian, but no such variation has been traceable in the present data. Four eruptions observed in longitudes lying between 55° and the limb have shown the red-shift to the extent required by the relationship in fig. 2.

There remains the probability that the red-shift is an effect of asymmetry in the contour arising from increased absorption on the violet side of the line. Let us suppose that the eruption is a centre from which matter capable of producing absorption is being expelled in all directions—there is, indeed, evidence of such expulsion in specific directions (*e.g.* the eruptive prominences). It is then at least likely that the amount of matter being expelled, and the absorption produced by it, will increase with the line-width and intensity of the eruption in the way that has been observed. Furthermore, if the expulsion took

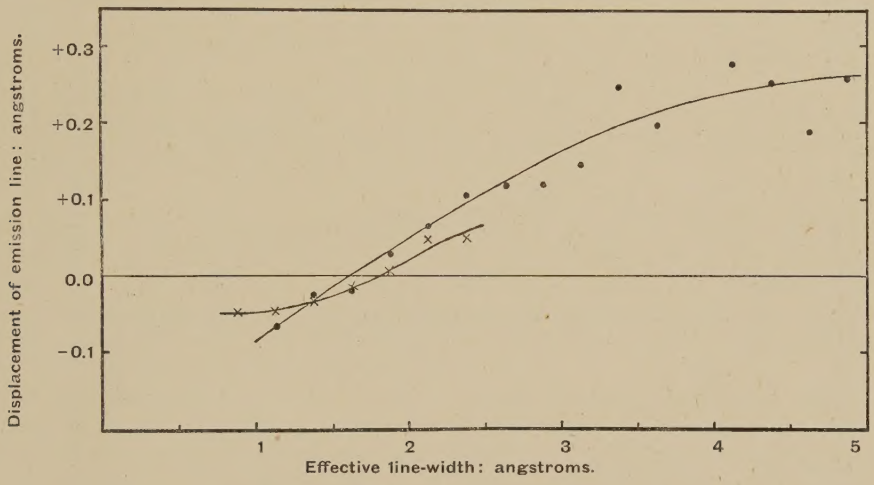


FIG. 2.—*Displacement of Hα emission line from centre of Hα absorption, plotted as a function of line-width: circles—in eruptions; crosses—in quiescent bright hydrogen.*

place uniformly within a solid angle of 180°, the absorption produced would be observable equally in all positions on the disk. Each eruption would then provide us with something like a nova phenomenon on a small scale. The further verification of this suggestion must await the collection of more numerous and improved data.

5. *Variations of Line-width in Eruptions.*—The curves of fig. 3 illustrate the growth and decay of line-width during the progress of 10 eruptions.* Fuller details of these will be found in Table II. The curves for those eruptions which have been fully observed show a rapid rise to maximum width, coinciding with the peak of visual intensity; this is followed by a steady but less rapid decline to the original value.

No. 1 is of special interest as showing an increase in line-width from 1·5 to 6·1 A., taking place in about 3 minutes and possessing an unusually sharp peak. Though of small extent (280 millionths), the visual intensity was judged greater than in any Class III eruption so far observed. Nos. 9 and 10, both Class III eruptions, show quite well-marked fluctuations following the time of maximum intensity, and these may perhaps be regarded as manifestations of the occurrence of secondary eruptions following the larger. But this behaviour is unusual, for the majority of eruptions appear to be quite

* It is understood that similar curves have been published by M. Waldmeier (*Zs. für Ap.*, 20, 46, 1940), but war-time restrictions have so far made it impossible to consult more than a brief summary of his original paper.

distinct and catastrophic events, each one showing in varying degree the same sequence of line-width and light variation.

6. *Lateral Motions of Eruptions.*—The proximity of stationary landmarks, like the

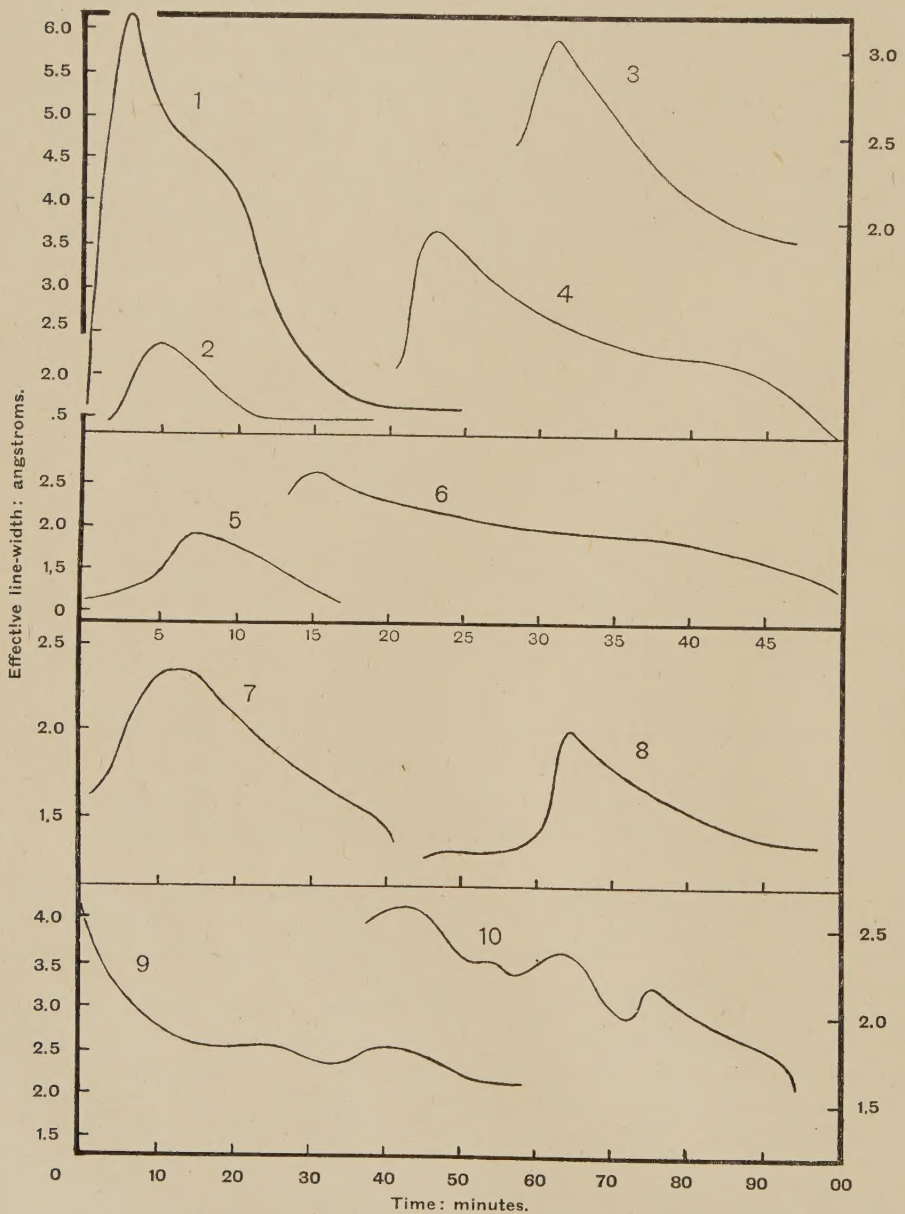


FIG. 3.—*Variation in width of the Hα emission line plotted against time during the course of ten bright eruptions. For details of each eruption see Table II.*

umbræ of adjacent sunspots, makes it possible to fix with some accuracy the co-ordinates of an eruption. This invariably begins with the sudden growth of a few brighter spots in an area of quiescent bright hydrogen. The really great eruptions frequently have the form of brilliant "ropes" or filaments many times greater in length than breadth. The brighter points and filaments are usually sharp and well-defined in the initial

TABLE II

Number of Eruption (see fig. 3)	Date	Time of initial Measure of Line-width, U.T.	Time of observed Maximum Intensity, U.T.	Position of Eruption		Importance (*d* denotes associated eruptive flocculi)
				Lat.	Long. from C.M.	
		h m	h m	°	°	
1	1942 Nov. 14	11 11	11 13	−13	W. 59	2*d*
2	,, Mar. 14	13 12	13 15	+ 3	E. 12	1*d*
3	,, June 26	14 28	14 30	+ 6	W. 58	1
4	,, Jan. 7	13 16	13 18	+10	W. 11	2*d*
5	1941 Oct. 7	13 35	13 43	+14	W. 6	1
6	,, June 6	16 44	16 45	+12	E. 34	3*d*
7	1942 Dec. 31	10 51	10 58	+ 3	W.24	1
8	,, Dec. 30	9 40	9 58	+ 2	W. 10	1
9	,, Feb. 28	12 47	...	+ 7	E. 4	3*d*
10	,, Nov. 7	10 27	10 26	−12	W. 64	3*d*

Notes

1. Area only 280 millionths, but of great brilliance.
4. See *B.A.A. Journal*, 52, 68, 1942.
9. See *The Observatory*, 64, 260, 1942. Eruption of long duration. Maximum intensity probably occurred shortly after $12^h\ 00^m$ U.T.

stages of the eruption, but once the greatest intensity is past a slow outward diffusion and spreading is observed without any movement as a whole. In 95 eruptions no case has been found where lateral movements could be measured with certainty. A few cases of apparent movement have been satisfactorily explained either by the outward diffusion, just mentioned, or by the fading of one part of the bright emission followed by an increase in brightness of an adjacent part.

Two critical cases may be mentioned, the Class III eruptions of 1940 March 27,* and 1942 February 28.† Both exhibited brilliant filaments and small points of *Hα* emission superimposed upon and partially obscuring the umbræ of a large spot-group. The former was under observation for 35 minutes and the latter intermittently for $2^h\ 40^m$—one of the longest eruptions on record. During these periods it was estimated that persisting lateral movements of the filaments as slow as 1 km./sec. would have been easily detected. We thus conclude that in the sample of chromospheric eruptions covered by this review there is no evidence for the existence of appreciable lateral movements in the bright emission.

* *B.A.A. Journal*, 50, 246, 1940. † *Loc. cit.*

A PHOTOGRAPHIC SURVEY OF GALACTIC CLUSTERS, IV: N.G.C. 6596, 6603, 6605, 6613, 6618, 6645, 6647, I.C. 4725, Tr. 33, Cr. 371, 469

George Alter

(Communicated by the Director, Norman Lockyer Observatory)

(Received 1943 January 19)

Summary

The region of this cluster group lies near the Small Sagittarius Cloud. Of the 11 star groups recorded in different catalogues as clusters 5 were considered not to be true clusters. The CI of some measured clusters agreed well, in the first determination, with the spectral types of their members; this, however, seems inconsistent with the fact that the clusters lie within a region where selective absorption has been found and some CE might be expected. Two explanations of this apparent contradiction are suggested, of which one has a greater probability and was used for the final results.

N.G.C. 6645 and Tr. 33 are shown, or suggested, to contain giants. The former contains a remarkable secondary condensation of 10 stars, the spatial density of which is larger than in any known galactic cluster. In the region of I.C. 4725 are two very red stars that may be situated in the neighbourhood of the Sun and one of which may be a variable.

The method of this investigation has been described in three earlier papers*, referred to later as A I, A II, A III. The photographs of the present group of clusters were taken on a clear night; order of exposures: cluster, NPS. The magnitudes of NPS show a mean deviation of $\pm 0^m.11$ on the photographic scale and of $\pm 0^m.15$ on the photovisual scale. The mean error of the CI is therefore $\pm 0^m.19$.

The group of clusters lies within a region which includes most of the Small Sagittarius Cloud. An inspection of Plate 7 of Ross-Calvert's *Atlas of the Milky Way* shows the complexity of the apparent star distribution at this place (6 cm. square just below the plate centre) †; it reveals also the difficulty of distinguishing, among the many large and small star condensations, between real condensations and those which are only effects of contrast against obscured neighbouring parts, and of deciding in some cases whether a condensation may be considered a galactic cluster, especially when the concentration is slight and the outline definition not clear. There is no convention on this point, and a recent investigation on the apparent structure of clusters ‡ showed that there is very little agreement among various cluster catalogues on the recognition of clusters in such cases. As a rule, this survey does not consider as galactic clusters condensations that have no appreciable concentration and boundary. Several objects of this kind within the region of this paper have been included in cluster catalogues (N.G.C. 6596, 6605, 6647, Cr. 469, 371), but will not be treated here in the same way as the more definitely recognizable clusters.

The results obtained for the latter present a further difficulty. These were derived from magnitudes determined from the measured diameters of the star images corrected for fog effect and an atmospheric extinction corresponding to the individual zenith distance of every cluster (this was necessary because of the large zenith distance of about 70°) and assuming the photographic coefficient to be double the photovisual. The mean CI were

* *M.N.*, 100, 387, 1940; *M.N.*, 101, 89, 1941; *M.N.*, 101, 298, 1941.

† See also E. v. d. Pahlen, *Z. f. Ap.*, 10, 111, Table 1, 1935.

‡ Not yet published.

N.G.C. 6613	6618	Tr. 33	I.C. 4725	N.G.C. 6645
$-0^m.15$	-0.06	$+0.10$	$+0.03$	$+0.02$

and no appreciable space reddening seemed to have affected them although the region (gal. long. about 340°) lies within, or next to, the largest aggregation of absorbing clouds within the galaxy.* Space reddening of B-type stars was established by Stebbins, Huffer and Whitford † within a region which partly coincides with the present one, and 36 stars of Table 3 of one of their later papers ‡ yield $+0^m.28$ as the mean colour excess of the B stars in the region of the cluster group. (Only 2 of the 36 stars lie, however, within individual cluster boundaries.) Baker and Kiefer § derived from star counting a total absorption of $1^m.0$ to $2^m.0$ for their region K, which includes nearly all these clusters.

There were two possible explanations for these contradictory results; either the clusters lie in regions where there is little or no absorbing matter, or the CI of the present investigation had a systematic error.

The first explanation finds little support from a visual inspection; only N.G.C. 6645 lies within a bright star cloud, while the surroundings of the others are comparatively dark and therefore suspected to be influenced by absorbing clouds.

As regards the second explanation, it is only an error in the assumption that the photographic atmospheric extinction is double the visual that might account for such a systematic error. Its influence is not very significant when the differences in altitude (cluster – NPS) are relatively small, but in the present case the zenith distance of the cluster group is about 70°. The assumption is based on a suggestion of various authors. || If we take, however, the photographic extinction coefficient of $0^m.24$ used by Lindsay and Bok ¶ and derived from Miss Leavitt's ** Arequipa plates and reduce it on account of the sea-level difference †† we get $0^m.31$ as the photographic extinction coefficient for this observatory, giving

$$\Delta m = 0^m.31 \ (\sec z - 1). \qquad (1)$$

At $z = 50°$ the difference between the final corrections from the two methods becomes appreciable (about $-0^m.04$), and at the zenith distances of the present cluster group it becomes $-0^m.28$, which happens to agree exactly with the mean CE of Stebbins, Huffer and Whitford, mentioned above. The presence of this CE seemed therefore to support the suggestion that a systematic error in the present results was caused by a wrong photographic extinction coefficient.

On the other hand, among the 18 HD stars distributed within the regions of the measured clusters, 10 stars (all situated in the cluster region of I.C. 4725) were found in this paper to have a negative CE ($-0^m.15$), 9 of them are found by the HD catalogue to have a CE of $-0^m.31$ and 6 of them, in Phyllis Hayford's ‡‡ Table II, a CE of $-0^m.37$. (The difference between the first and the two other CE is due to a systematic difference: the m_{pv} of this paper are about $0^m.17$ smaller than the HD values, used also by Hayford.)

This unsatisfactory situation was also not improved by considering the ranges of distance moduli, which sometimes indicate the presence of CE (see A II). In the present case the observed mean CI coincided generally with the true ones, and therefore the observed $(m - M)$ ranges agreed generally with the expected ones.

On the whole, however, the second explanation, supported by independent results on absorption and implying the use of the Arequipa coefficient, is more probable than the first one, and the m_p were therefore redetermined with the coefficient given in formula (1). The values of M are derived (as with N.G.C. 663 in A II) from the CI corrected by the CE. The values for distances and linear diameters, determined with the assumption of absorption, are followed (in brackets) by values which assume no absorption.

Finally it may be mentioned that the investigations of clusters published in A I to

* *M.N.*, 102, 211, 1942. † *Ap. J.*, 90, 222, 1939. ‡ *Ap. J.*, 91, 20, 1940.
§ *Ap. J.*, 96, 224, 1942. || See *Hdb. d. Ap.*, 2, 490, 1931. ¶ *H.A.*, 105, 261, 1937.
** *H.A.*, 74, 255, 1913. †† See *Hdb. d. Ap.*, 2, 181, form. 41, 1929. ‡‡ *L.O.B.*, 16, 63, 1932.

A III are not appreciably affected by the change of the photographic coefficient because the zenith distances are too small.

N.G.C. 6603 (M 24)

R.A. = $18^h\ 12^m.6$, Dec. = $-18°\ 27'$ (1900·0)

This cluster is mentioned in the catalogues of Shapley, Trumpler and Collinder, is investigated by Raab *, and a good picture of it is reproduced in Roberts's *Celestial Photographs*, **2**, Plate 3 (1899). It lies within the Small Sagittarius Cloud, one of the richest Milky Way regions. It consists of a very dense aggregation of faint stars, and this makes it impossible to use the Sidmouth plates for magnitude determination; the overlapping star images are blotted into grey patches without any distinguishable edges. Thus even a determination of co-ordinates seemed unreliable and was not attempted.

The star counting gave 130 stars as the total number within the region. The distribution of stars within the cluster region and its surroundings results in a background density of 13,000 stars per square degree and 84 cluster stars within a cluster region of 4′ diameter. The brightest among them are of about 13^m. The distances estimated in the various catalogues on different assumptions range from about 3 to 5 kpcs. If, however, J. Herschel's suggestion is accepted that the cluster is part of the cloud, then its distance would be much smaller. Wallenquist † found the cloud distance (uncorrected for absorption) to be 1300 parsecs, and Stebbins, Huffer and Whitford ‡ derive from their investigation a true distance of 1100 parsecs.

N.G.C. 6613 (M 18)

R.A. = $18^h\ 14^m.1$, Dec. = $-17°\ 10'$ (1900·0)

This cluster is mentioned in the catalogues of Shapley, Trumpler and Collinder and investigated by Trumpler and Carol Anger Rieke.§

The star counting gave 69 stars as the total number within the region. The distribution of stars within the cluster region and its surroundings results in a background

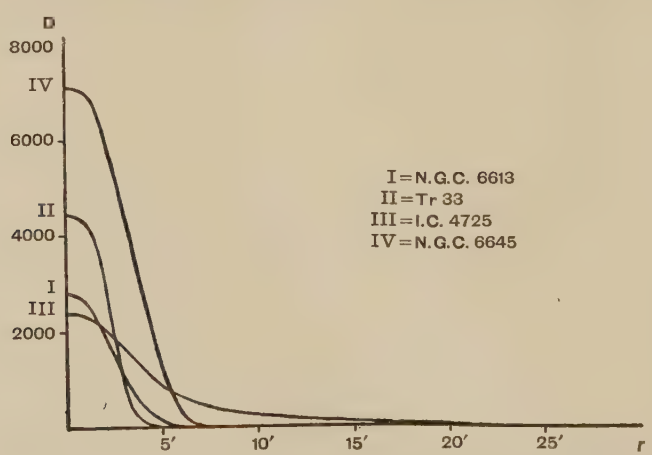


FIG. 1.—*Star density per square degree above background density.*

density of 2200 stars per square degree and 23 cluster stars within a cluster region of 9′·8 diameter. Fig. 1 gives the diagram of the star density per square degree within the cluster region.

* *Lund. Medd.*, Ser. II, 28, 1922.

† *Medd. Upsala*, 74, 1938.

‡ *Ap. J.*, 90, 223, 1939.

§ *H.C.*, 397, 1935.

TABLE I

Co-ordinates, Magnitudes and Distance Moduli of 36 Stars (N.G.C. 6613)

Zero point = HD 168418, unit = 1′·0, X = west, Y = south
Co-ordinates of the assumed cluster centre $X = +4·5$, $Y = +4·6$

Star	X	Y	m_p	m_{pv}	$m-M$	Star	X	Y	m_p	m_{pv}	$m-M$
37	+4·6	+8·4	11·31	10·59	6·2	10	+7·4	+8·4	10·86	10·94	13·2
46	3·9	5·7	10·59	10·11	8·3	67	1·4	4·2	12·94	12·84	13·6
9	7·5	7·1	13·64	12·83	8·8	45	4·0	6·1	9·96	10·14	14·1
50	3·1	3·3	9·83	9·61	8·8	2	8·4	5·1	13·04	13·00	14·7
34	4·7	1·1	13·05	12·61	10·0	13	7·0	2·9	13·77	13·74	15·5
1 *	8·6	2·7	11·30	11·05	10·1	33	5·2	8·4	12·37	12·44	15·7
52	3·0	5·2	12·98	12·41	10·1	3	8·2	7·6	12·79	13·01	17·0
39	4·5	4·9	12·45	12·23	11·2	26	5·6	2·9	12·72	13·01	17·0
14 †	6·8	2·1	10·74	10·65	11·7	12	7·2	6·7	12·87	13·14	17·1
49	3·4	4·9	10·69	10·63	11·9	15	6·9	7·8	12·69	13·07	17·1
68	1·2	6·9	10·35	10·30	11·9	6	7·8	4·4	12·82	13·42	17·4
43 ‡	4·1	3·4	10·02	10·00	12·0	30	5·4	8·1	13·18	13·45	17·4
29 §	5·4	4·2	9·01	9·10	12·4	35	4·6	0·3	13·92	14·00	17·4
4	8·0	5·2	14·01	13·73	12·5	27	5·7	8·6	13·36	13·63	17·6
18	6·6	7·2	10·69	10·67	12·7	38	4·6	7·6	12·43	13·55	17·6
28	5·5	5·9	10·74	10·72	12·7	41	4·3	5·2	13·36	13·58	17·6
7	7·7	5·7	14·43	14·14	12·9	44	4·0	0·5	13·45	13·68	17·7
48	+3·4	+6·4	14·27	14·04	13·1	21	+6·3	+8·7	13·54	14·01	18·0

* = HD 168303. † = 168327. ‡ = 168368. § = 168352.

Table I gives the distribution of $m-M$. The mean distance modulus of the cluster group is 12·267, corresponding to

$$r = 2070\ (2840) \pm 170 \text{ parsecs}, \qquad d = 9'·8 = 5·9\ (8·1) \text{ parsecs}.$$

The results of the previous investigations are

	Distance	Diameter	
	Parsecs	′	Parsecs
Shapley	1200–1910	12·0	6·7
Trumpler	1780	7·0	4·1
Collinder	2400	7·5	5·3
Anger Rieke	1380	...	...

The difference between this distance and that of Trumpler is probably due to the use by Trumpler of only 3 stars for the determination, and of Anger Rieke to the uncertainty of observation, indicated by parentheses with most of the stars used in her diagram.

The number of cluster stars, after correction for the background stars, is 26 against 23 got statistically; the dispersion is $\sigma = \pm 5$. The magnitude distribution is given in the following table:—

m_{pv}	9	10	11	12	13	> 13·5
Number of stars	1	3	6	2	2	12

N.G.C. 6618 *a*

R.A. = $18^h\ 15^m·3$, Dec. = $-16°\ 4'$ (1900·0)

Shapley records under N.G.C. 6618 and Collinder under 377 a cluster whose co-ordinates agree with those of the original N.G.C. 6618 which is the so-called Omega

Nebula. (A historical survey of the nebula is given by Duncan.*) There is, however, a cluster-like group of stars 10′ north following the nebula (see *L.O. Publ.*, 8, 58) which investigation showed to be a real cluster. This group is also mentioned as possibly a cluster by Hubble †, but placed mistakenly at the nebula's southern border. The group covers an elliptic area of about 15′ length and 7′ width, orientation *sp* – *nf*, and comprises 49 stars down to the photovisual magnitude 14. No condensation of the faint stars being found, the usual statistical treatment was not done and the number of cluster members was determined by the CI method. (A rough comparison with a nearby region of the same area showed a star surplus of 17 stars brighter than 14^m.)

Table II gives the contribution of $m - M$. The mean distance modulus of the cluster group is 11·971, corresponding to

$$r = 1840\ (2470) \pm 130 \text{ parsecs}, \qquad d = 7' \times 15' = 3{\cdot}6\ (5{\cdot}1) \times 7{\cdot}5\ (10{\cdot}8) \text{ parsecs}.$$

The results of the previous investigations were:

	Distance	Diameter	
	Parsecs	′	Parsecs
Shapley	1050–1660	...	...
Collinder	1000	8 × 15	2·3 × 4·4

TABLE II

Co-ordinates, Magnitudes and Distance Moduli of 41 Stars (N.G.C. 6618 a)

Zero point = star 30 = HD 168585, unit = 1′·0, X = west, Y = south
Co-ordinates of the assumed cluster centre $X = +1{\cdot}3$, $Y = +1{\cdot}6$

Star	X	Y	m_p	m_{pv}	$m - M$	Star	X	Y	m_p	m_{pv}	$m - M$
12	+4·1	+2·8	11·08	10·74	9·4	5	+5·9	+2·0	14·01	13·76	12·8
3	+6·5	+5·0	11·29	10·89	9·5	36	−1·5	−1·4	12·91	12·77	13·0
25	+0·9	+4·3	11·62	11·16	9·5	8	+5·1	+2·5	13·19	13·05	13·1
41	−2·3	−1·1	11·36	10·98	9·6	15	+2·6	+4·8	13·63	13·43	13·1
22 *	+1·2	+4·2	10·76	10·54	9·7	19	+1·8	+4·3	11·91	11·84	13·1
18	+1·9	+0·7	10·42	10·24	9·9	6	+5·6	+1·3	10·56	10·59	13·3
28	+0·5	−1·4	12·17	11·69	9·9	9	+4·5	+1·8	10·78	10·80	13·6
30 †	0·0	0·0	9·79	9·70	10·6	42	−2·6	0·0	14·01	13·85	13·8
23	+0·9	−0·9	12·05	11·79	10·7	27	+0·5	−3·4	12·75	12·70	14·2
34	−0·6	+5·4	12·02	11·79	10·9	49	−4·2	−2·8	10·19	10·32	14·3
31	−0·4	−3·3	12·05	11·85	11·3	11	+4·1	+1·8	14·01	13·79	14·3
32	−0·4	−0·6	12·88	12·57	11·4	48	−3·8	+0·9	10·22	10·38	14·4
33	−0·5	−1·3	13·87	13·39	11·6	1	+6·9	+5·5	11·84	11·91	15·0
24	+0·9	+2·4	13·87	13·45	12·1	45	−3·4	−0·7	12·28	12·31	15·0
39	−1·8	−4·5	12·44	12·27	12·1	4	+6·3	+2·2	11·74	11·92	15·9
17	+2·0	+0·3	12·31	12·16	12·2	21	+1·2	+3·4	12·07	12·20	16·2
20	+1·6	+1·3	13·77	13·46	12·3	2	+6·9	+4·6	12·16	12·49	16·5
13	+4·2	+4·8	13·94	13·64	12·4	7	+5·6	+4·7	12·70	12·86	16·9
14	+2·8	+2·2	14·23	13·84	12·4	10	+4·3	+2·0	12·81	13·03	17·0
35	−1·2	0·0	13·94	13·64	12·4	26	+0·6	−1·9	13·01	13·17	17·2
38	−1·8	−5·1	14·01	13·67	12·4	37	−1·7	−4·2	12·87	13·18	17·2
16	+2·3	+5·5	11·30	11·23	12·5						

* = HD 168569. † = 168585.

The number of cluster stars, after correction for the background stars, is 20 against 17 got by estimation; the dispersion is $\sigma = \pm 4$. The magnitude distribution is given in the following table:—

m_{pv}	10	11	12	13	14
Number of stars	3	7	2	5	3

* *Ap. J.*, 51, 9, 1919. † *Ap. J.*, 56, 185, 1922.

Tr. 33

R.A. = $18^h\ 18^m.8$, Dec. = $-19°\ 44'$ (1900·0)

This cluster is mentioned in the catalogues of Trumpler and Collinder, and is noted in Barnard's *Milky Way Atlas* (Plate 32) as "a small group, 6′ in diameter, of a few bright stars."

The star counting gave 51 stars as the total number within the region. The distribution of stars within the cluster region and its surroundings results in a background density of 2523 stars per square degree, and in 25 cluster stars within a cluster region of 6′·6 diameter. Fig. 1 gives the diagram of the star density per square degree within the cluster region.

Table III arranges the stars in two different ways. In the first part the absolute magnitudes were taken, as usual, from the main sequence. Here appear two ranges in the distance modulus (similar to N.G.C. 659 in AII, p. 96), the first including the stars between 40 and 2, the second between 9 and 47, the two being divided by the jump of $m-M$ between 2 and 9. As in the previous case mentioned we may suspect the first group to represent giants, and the second part of Table III gives the rearrangement where the suspected giant stars fit very well in a common range (which, on account of the small star number, is smaller than expected). A confirmation of the assumption made above can of course be yielded only by a spectral investigation of the cluster which is not yet available.

TABLE IIIA

Co-ordinates, Magnitudes and Distance Moduli of 23 Stars (Tr. 33)

Zero point = star 15 = HD 169274, unit = 1′·0, X = east, Y = north
Co-ordinates of the assumed cluster centre $X = +1.3$, $Y = +0.6$

Star	X	Y	m_p	m_{pv}	$m-M$	Star	X	Y	m_p	m_{pv}	$m-M$
15*	0·0	0·0	9·34	8·03	2·1	27	+1·5	+2·0	14·01	13·79	13·0
40	+2·8	+1·2	11·93	11·03	6·5	43	+3·2	−1·8	12·82	12·77	14·3
18	+0·5	+0·8	9·80	9·22	6·8	4	−1·2	+0·3	14·08	14·03	15·5
23	+1·0	+1·8	10·15	9·59	7·3	26	+1·4	−0·3	11·51	11·67	15·7
19	+0·7	+3·6	9·66	9·37	8·2	6	−1·0	−0·2	14·17	14·14	16·0
2	−1·9	−0·4	11·25	10·76	8·9	3	−1·6	−0·4	13·84	13·87	16·6
9	−0·7	+2·1	13·85	13·24	10·6	21	+0·8	+2·5	14·00	14·03	16·7
38	+2·6	+1·8	12·29	11·93	10·6	33	+1·8	−2·3	12·88	13·00	16·7
8	−0·9	−1·7	12·83	12·55	11·3	28	+1·5	−1·6	13·56	13·71	17·6
46	+3·7	+1·1	13·10	12·79	11·6	24	+1·0	+2·4	13·84	14·17	18·2
14	0·0	+2·0	13·45	13·13	11·9	25	+1·3	−1·2	14·00	14·18	18·2
47	+3·7	−0·9	13·52	13·15	11·9						

* = HD 169274

TABLE IIIB

Star	$m-M$	Star	$m-M$	Star	$m-M$	Star	$m-M$	Star	$m-M$
15	2·1	23	9·6	2	10·8	46	11·6	27	13·0
18	9·2	9	10·6	40	11·0	14	11·9	43	14·3
19	9·4	38	10·6	8	11·3	47	11·9	etc.	etc.

The mean distance modulus of the cluster group is 10·718, corresponding to

$$r = 1160\ (1390) \pm 90 \text{ parsecs}, \qquad d = 6'\cdot6 = 2\cdot2\ (2\cdot7) \text{ parsecs}.$$

The results of the previous investigations were:

	Distance	Diameter	
	Parsecs	′	Parsecs
Trumpler	1870	5·5	3·0
Collinder	2970	5·0	4·3

The number of cluster stars, after correction for the background stars, is 20 against 25 got statistically; the dispersion is $\sigma = \pm 5$. Among the cluster stars may be 4 to 5 giants. The magnitude distribution is given in the following table:—

m_{pv}	9	10	11	12	13	> 13·5
Number of stars	2	1	2	2	2	11

I.C. 4725 (M 25)

R.A. = $18^h\ 25^m.8$, Dec. = $-19°\ 11'$ (1900·0)

This cluster is mentioned in the catalogues of Shapley, Trumpler and Collinder and investigated by Raab, Trumpler, Hayford and Anger Rieke.

There is some disagreement in the co-ordinates in different papers; the right co-ordinates are given above.

As in N.G.C. 6618 *a*, the stars fainter than 14^m show no condensation. The counting was therefore restricted to the brighter ones. It gave 183 stars as the total number within the region. The distribution of stars within the cluster region and its surroundings shows a background density of 352 stars per square degree and 102 cluster stars within a cluster region of 32′·8 diameter. Fig. 1 gives the diagram of the star density per square degree within the cluster region.

Table IV gives the distribution of $m - M$. The range of the distance modulus (7·2) exceeds by 3 the range expected according to A II, but this is not due to a CE, because M was corrected on account of the CE, as mentioned in the introduction. The average of the true CI (+0·03) agrees very well with the spectral range B4 to A5 of 47 stars, observed by Trumpler. The explanation is given by the large number of CI-stars. The expected range is derived from the mean errors of CI and M. With the increase of the number of stars concerned there is a simultaneous, although slower, increase of the number of such stars whose errors of CI and M exceed the mean values, and these stars extend the range. It will be seen that the star frequency for 1·0 step of the distance modulus between 8·7 and 12·7 (the expected range) is about 22, while it is about half this value in the external parts of the range.

Among the stars listed are two double stars: 103 is identical with β 966 (as established elsewhere *) and 126 with Rossiter 3186.† Knowing the m_{pv} and spectral types of 103's components and its integrated m_p, the individual m_p of the components can be computed; a splitting of 103 into its two parts would not move them outside the range. (The respective $m - M$ are 9·6 and 10·0.) No spectral types of Rossiter 3186 are known, but its position in the $(m - M)$-scale makes it improbable that a splitting would move both components within the range.

The distance modulus of star 171 (the first star of Table IV) is put down as 0± because of the large CI ($+2^m.73$) for which no M was available. The star seems to be in the neighbourhood of the Sun. The same may be true with star 128 ($X = -2\cdot8$, $Y = -8\cdot1$), which is not in Table IV because only a trace of its image was found on the

* *The Observatory*, to be published later.
† *Mem. R.A.S.*, 65, 63, 1936.

TABLE IV

Co-ordinates, Magnitudes and Distance Moduli of 167 Stars (I.C. 4725)

Zero point = star 117 = HD 170764, unit = 1′·0, X = west, Y = north
Co-ordinates of the assumed cluster centre $X = +1{\cdot}9$, $Y = +0{\cdot}5$

Star	X	Y	m_p	m_{pv}	$m-M$	Star	X	Y	m_p	m_{pv}	$m-M$
171	−10·8	+ 0·8	13·54	10·79	0 ±	70	+ 3·8	− 2·7	11·02	10·62	9·1
141 *	− 4·7	+ 0·1	8·94	7·72	1·8	114	+ 0·1	−10·7	10·23	9·92	9·1
117 †	0·0	0·0	8·16	7·10	1·9	115	0·0	− 9·5	12·30	11·72	9·1
162 ‡	− 8·6	+ 9·3	8·84	7·70	2·1	48	+ 5·9	+ 2·3	11·60	11·13	9·2
28	+10·1	−12·4	13·94	11·85	2·3	97	+ 2·0	+ 2·1	10·64	10·29	9·2
151	− 6·8	+ 0·8	12·66	10·92	3·3	134	− 4·2	−13·0	9·88	9·63	9·2
178	−13·7	− 1·8	13·25	11·51	3·8	161	− 8·7	+ 5·4	10·42	10·10	9·2
84	+ 2·9	− 2·6	11·90	10·75	5·2	71	+ 3·8	+ 1·3	11·54	11·09	9·3
100	+ 1·4	+ 8·2	10·62	9·71	5·2	18	+11·0	+ 1·3	10·31	10·03	9·4
33 §	+ 8·1	+13·4	7·88	7·43	5·6	62	+ 4·8	− 6·6	12·23	11·72	9·5
126	− 2·1	+ 1·9	10·52	9·70	5·7	94*a*	+ 2·2	− 3·1	9·47	9·29	9·5
98	+ 1·7	− 2·1	10·94	10·07	5·8	105	+ 0·6	− 5·0	11·27	10·88	9·5
94	+ 2·2	− 2·9	9·16	8·58	6·0	136	− 4·2	− 3·2	12·69	12·04	9·5
46	+ 5·9	− 6·8	10·24	9·54	6·1	45	+ 5·9	−11·3	9·99	9·77	9·6
170	−10·7	+ 1·5	10·79	10·01	6·2	54	+ 5·4	− 7·5	12·41	11·88	9·6
158	− 7·8	−11·0	12·54	11·49	6·3	123	− 2·0	+ 9·3	13·71	13·02	9·7
104	+ 0·7	− 2·8	9·38	8·84	6·4	49	+ 5·9	+10·0	10·08	9·88	9·9
147	− 6·4	−10·1	13·01	11·90	6·5	73	+ 3·7	− 4·6	10·39	10·17	10·0
85	+ 2·7	− 5·7	12·90	11·85	6·7	144	− 5·7	+ 3·4	12·23	11·78	10·0
89	+ 2·6	−14·1	11·20	10·45	6·8	13	+12·1	− 0·9	11·22	10·91	10·1
179	−13·7	− 2·3	11·85	11·01	6·9	87	+ 2·8	+ 0·6	9·64	9·51	10·1
138	− 4·5	+ 7·3	12·15	11·30	7·1	95	+ 2·1	+ 0·9	9·99	9·84	10·2
56	+ 5·4	+ 3·8	11·02	10·37	7·3	1	+17·3	+ 5·3	9·84	9·71	10·3
129	− 2·8	− 2·6	10·47	9·92	7·4	6	+14·0	− 1·4	11·75	11·40	10·3
155	− 7·4	+ 9·1	9·78	9·32	7·4	21	+10·8	− 2·1	12·47	12·04	10·3
29	+ 9·1	+ 8·9	12·57	11·72	7·5	107	+ 0·5	+ 6·7	13·91	13·27	10·3
163	− 8·9	−10·2	9·77	9·32	7·5	137	− 4·3	+ 6·7	13·91	13·27	10·3
80	+ 3·2	+ 2·7	12·78	11·91	7·6	31	+ 8·2	+ 4·3	10·68	10·47	10·4
39	+ 7·2	+ 5·6	10·61	10·04	7·7	67	+ 4·6	− 8·5	10·59	10·39	10·4
61 ‖	+ 4·8	+ 7·9	9·60	9·20	7·7	24	+10·4	− 7·0	11·11	10·86	10·5
72 ¶	+ 3·8	+ 2·2	8·53	8·26	7·8	159 ‖‖	− 8·2	+ 1·5	9·51	9·44	10·5
172	−11·0	+10·0	10·84	10·28	7·8	10	+12·5	+ 9·2	12·87	13·28	10·6
12	+12·1	− 2·5	12·82	12·02	8·1	102	+ 1·1	+ 0·5	10·27	10·13	10·6
120 **	− 1·2	+12·6	9·23	8·94	8·2	35	+ 7·9	−11·2	12·41	12·04	10·7
148	− 6·5	+ 2·6	10·23	9·83	8·3	44	+ 6·2	+ 3·3	13·91	13·30	10·7
41 ††	+ 6·9	− 2·0	8·29	8·14	8·5	176	−12·9	+ 2·9	13·68	12·26	10·7
96	+ 2·1	+ 0·4	10·17	9·79	8·5	43	+ 6·1	+12·0	11·27	11·04	10·8
140 ‡‡	− 4·6	− 9·1	9·17	8·91	8·5	106	+ 0·6	− 4·7	11·87	11·57	10·8
156	− 7·7	0·0	12·26	11·62	8·6	63	+ 4·8	− 5·5	12·56	12·18	10·9
20	+10·9	+ 7·8	12·53	11·87	8·7	92	+ 2·4	+ 0·5	9·64	9·58	10·9
47	+ 6·0	− 1·5	9·77	9·47	8·7	27	+10·3	+ 7·6	13·06	12·64	11·0
83	+ 2·9	− 7·8	9·84	9·54	8·7	99	+ 1·7	+ 1·0	10·18	10·08	11·0
173	−11·1	+ 4·1	11·12	10·65	8·7	180	−14·0	− 2·6	13·51	13·03	11·0
118	− 0·4	− 3·1	11·96	11·39	8·8	57	+ 5·2	+ 1·4	12·20	11·89	11·1
157	− 7·8	+ 2·5	12·41	11·77	8·8	116	− 0·1	− 7·9	11·25	11·04	11·1
167	− 9·8	− 2·4	11·15	10·69	8·8	69	+ 4·0	− 1·1	12·03	11·75	11·2
143 §§	− 5·5	− 5·6	9·25	9·04	8·9	165	− 9·6	+ 9·0	12·84	12·48	11·2
103	+ 1·0	− 0·4	9·63	9·38	9·0	82	+ 2·9	+ 5·4	12·26	11·97	11·3
169	−10·6	+ 4·8	9·88	9·59	9·0	3	+15·5	+ 1·4	13·91	13·43	11·4
5	+13·9	+10·0	12·53	11·92	9·1	38	+ 7·2	+ 6·0	12·64	12·34	11·4

* = HD 170820. † = 170764 = U Sgr. ‡ = 170886. § = 170657. ‖ = 170681. ¶ = 170719.
** = 170763. †† = 170682. ‡‡ = 170836. §§ = 170835. ‖‖ = 170860.

TABLE IV—*continued*

Star	X	Y	m_p	m_{pv}	$m-M$	Star	X	Y	m_p	m_{pv}	$m-M$
52	+ 5·5	+16·1	9·83	9·81	11·4	142	− 4·7	+ 4·7	13·23	13·06	13·4
121	− 1·3	0·0	11·44	11·25	11·4	177	−13·2	− 2·2	13·14	12·99	13·4
109	+ 0·2	+15·5	12·53	12·24	11·5	166	− 9·6	+10·6	13·17	13·04	13·6
15	+12·0	+ 6·2	12·26	12·01	11·6	7	+13·9	+ 2·7	12·40	12·36	13·8
26	+10·4	0·0	10·41	10·35	11·6	74	+ 3·4	−13·9	13·68	13·52	13·9
37	+ 7·7	+ 5·4	13·31	12·94	11·6	76	+ 3·4	+ 4·5	13·91	13·73	13·9
68	+ 4·2	+11·6	12·23	11·97	11·6	113	+ 0·1	+ 4·2	11·69	11·70	14·0
145	− 5·8	− 3·4	11·84	11·64	11·6	181	−14·4	+ 0·5	13·14	13·05	14·1
23	+10·4	−13·0	12·12	11·89	11·7	124	− 1·9	+13·4	12·10	12·10	14·2
59	+ 5·1	− 7·0	11·93	11·73	11·7	16	+11·6	+ 1·8	11·98	11·99	14·4
135	− 4·2	− 3·7	10·58	10·52	11·7	34	+ 8·0	− 3·6	13·67	13·57	14·5
66	+ 4·6	+15·9	12·33	12·09	11·8	130	− 2·8	+ 5·5	13·61	13·53	14·5
108	+ 0·2	+14·0	12·20	11·97	11·8	101	+ 1·3	+ 7·8	12·72	12·74	15·2
65	+ 4·7	+10·2	11·81	11·64	11·9	91	+ 2·5	−12·6	11·13	11·28	15·3
81	+ 2·9	+ 1·3	12·77	12·49	11·9	93	+ 2·2	+ 9·9	12·46	12·50	15·4
53	+ 5·5	− 8·4	10·67	10·64	12·1	25	+10·4	− 4·9	12·61	12·94	15·9
122	− 1·6	+15·8	12·03	11·85	12·1	50	+ 5·8	+ 8·8	11·31	11·89	15·9
9	+13·1	− 7·3	10·33	10·24	12·2	139	− 4·5	− 7·3	12·63	12·70	16·2
90	+ 2·5	+ 1·8	12·89	12·63	12·2	110	+ 0·3	− 3·2	12·99	13·05	16·4
127	− 2·7	+ 3·7	13·91	13·53	12·2	42	+ 6·6	− 9·7	13·05	13·12	16·5
174	−11·1	+ 9·3	12·53	12·32	12·2	149	− 6·6	− 4·9	12·55	12·67	16·7
55	+ 5·4	+ 1·8	12·23	12·05	12·3	77	+12·8	+ 7·3	12·60	12·78	16·8
131	− 2·8	+ 6·1	13·23	12·95	12·4	79	+ 3·2	− 2·7	12·60	12·90	16·9
51	+ 5·6	− 2·2	13·61	13·32	12·6	111	+ 0·2	− 4·4	13·30	13·37	16·9
64	+ 4·8	− 1·6	13·61	13·32	12·6	2	+15·7	+ 3·9	12·65	13·02	17·0
175	−11·3	− 3·1	13·51	13·23	12·6	40	+ 7·1	− 3·7	12·99	13·15	17·1
32	+ 8·1	+ 8·8	13·61	13·34	12·7	75	+ 3·5	− 7·0	13·05	13·16	17·2
88	+ 2·6	+ 3·3	12·61	12·44	12·7	86	+ 2·8	− 4·7	12·54	13·19	17·2
112	+ 0·2	− 3·0	13·72	13·44	12·8	14	+12·0	+ 2·7	13·05	13·34	17·3
58	+ 5·1	+ 9·3	13·61	13·35	12·9	133	− 3·5	−12·2	13·13	13·38	17·4
168	−10·4	+ 2·7	12·81	12·64	12·9	132	− 2·9	−11·9	13·30	13·46	17·5
36	+ 7·8	−10·6	12·92	12·75	13·1	119	− 0·5	− 3·5	13·39	14·00	18·0
150	− 6·7	−12·1	13·00	12·83	13·1	152	− 6·8	+10·7	13·39	13·99	18·0
146	− 5·9	−13·4	13·91	13·66	13·3						

photographic plate ($> 14^m.0$) although $m_{pv} = 11.90$. Moreover, an image of this star can be found on the Franklin-Adams Charts, where its m_p is about $13^m.2$, so that the star may well be a red variable.

The mean distance modulus of the cluster group is 10·677, corresponding to

$$r = 1130\ (1370) \pm 100 \text{ parsecs}, \qquad d = 32'.8 = 10.8\ (13.1) \text{ parsecs}.$$

The results of the previous investigations were:

	Distance	Diameter	
	Parsecs	′	Parsecs
Shapley	570–910	40	10·6
Trumpler	980	35	10·0
Collinder	910	30 × 25	7·3
Anger Rieke	690	...	...

The number of cluster stars, after correction for the background stars, is 98 against 102 got statistically; the dispersion $\sigma = \pm 10$. The magnitude distribution is given in the following table:—

m_{pv}	8	9	10	11	12	13	14
Number of stars	2	9	29	15	22	17	4

TABLE V

Co-ordinates, Magnitudes and Distance Moduli of 81 *Stars* (*N.G.C.* 6645)

Zero point = HD 171029, unit = 1′·0, X = west, Y = south
Co-ordinates of the assumed cluster centre $X = +8{\cdot}9$, $Y = +3{\cdot}6$

Star	X	Y	m_p	m_{pv}	$m-M$	Star	X	Y	m_p	m_{pv}	$m-M$
177	+ 7·0	− 0·5	11·29	9·80	2·9	54	+11·4	+ 0·4	12·72	12·46	12·1
220	+ 4·4	+ 1·7	12·09	10·50	3·1	105	+ 9·4	+ 0·9	13·27	12·96	12·2
42	+12·1	+ 3·8	11·70	10·26	3·6	64	+11·1	+ 5·4	13·62	13·28	12·3
81	+10·6	+ 6·5	12·43	10·89	3·8	93	+10·0	+ 1·4	12·92	12·67	12·3
156	+ 7·8	+ 1·1	12·35	10·89	4·1	117	+ 9·1	+ 1·8	12·72	12·49	12·3
211	+ 5·4	+ 6·0	12·84	11·27	4·1	195	+ 6·4	+ 3·2	13·84	13·48	12·3
65	+11·1	+ 4·5	12·59	11·09	4·2	32	+12·7	+ 5·3	12·70	12·49	12·4
127	+ 8·8	+ 5·5	11·01	9·89	4·4	106	+ 9·4	+ 2·2	14·17	13·78	12·4
147	+ 8·2	+ 3·7	12·45	11·27	5·5	212	+ 5·3	+ 2·9	13·54	13·23	12·4
55	+11·3	+ 0·8	12·08	11·04	5·8	86	+10·4	+ 4·1	13·44	13·16	12·6
179	+ 7·1	+ 2·7	12·73	11·58	6·0	118	+ 9·0	− 0·2	14·27	13·91	12·7
180	+ 7·0	+ 2·0	11·55	10·63	6·0	125	+ 8·8	+ 4·8	12·49	12·33	12·7
203	+ 5·9	+ 1·9	10·54	9·84	6·4	160	+ 7·8	+ 7·7	14·17	13·82	12·7
185	+ 7·0	+ 8·3	12·31	11·36	6·7	128	+ 8·6	− 0·3	12·66	12·49	12·8
133	+ 8·6	+ 1·0	12·64	11·72	7·0	184	+ 7·0	+ 3·2	12·72	12·56	13·0
154	+ 8·0	+ 7·8	12·70	11·82	7·4	61	+11·0	− 1·2	13·62	13·38	13·1
213	+ 5·1	+ 1·2	13·11	12·16	7·5	60	+11·2	+ 5·7	12·94	12·78	13·2
111	+ 9·3	+ 2·8	12·00	11·34	8·2	172	+ 7·3	+ 0·7	12·87	12·72	13·2
114	+ 9·0	− 0·9	14·11	13·12	8·2	170	+ 7·5	+ 1·6	13·55	13·35	13·5
109	+ 9·3	+ 3·5	12·44	11·76	8·6	129	+ 8·7	+ 3·6	13·01	12·90	13·7
41	+12·2	+ 6·8	12·29	11·67	8·8	36	+12·5	+ 3·3	13·52	13·36	13·8
5	+14·0	+ 4·3	12·57	11·97	9·2	205	+ 5·9	+ 6·5	12·89	12·80	13·8
199	+ 6·1	+ 4·7	11·54	11·15	9·8	7	+13·9	+ 3·9	14·02	13·83	13·9
224	+ 4·2	+ 0·9	12·95	12·37	9·8	96	+ 9·8	− 0·4	13·42	13·29	13·9
151	+ 8·2	+ 7·9	12·66	12·17	10·0	100	+ 9·7	+ 1·8	14·25	14·05	14·0
108	+ 9·3	+ 2·6	12·38	11·95	10·3	131	+ 8·6	+ 0·8	13·84	13·67	14·0
190	+ 6·6	+ 3·4	14·02	13·47	11·0	143	+ 8·2	+ 0·2	13·92	13·84	14·8
176	+ 7·2	+ 4·5	12·93	12·55	11·3	113	+ 9·1	+ 0·2	13·74	13·68	14·9
191	+ 6·5	+ 1·3	12·56	12·24	11·3	229	+ 3·7	+ 2·8	12·86	12·86	15·1
110	+ 9·3	+ 4·9	12·72	12·38	11·4	57	+11·1	+ 2·8	13·61	13·64	16·3
95	+ 9·8	+ 5·5	13·84	13·38	11·5	120	+ 9·1	+ 6·0	13·41	13·35	16·3
97	+10·1	+ 7·0	13·27	12·88	11·5	76	+10·8	+ 5·0	12·56	12·64	17·0
137	+ 8·6	+ 5·8	12·29	12·02	11·5	173	+ 7·3	+ 3·0	13·18	13·28	17·3
144	+ 8·1	− 1·9	13·42	13·01	11·5	9	+13·7	+ 1·4	13·11	13·41	17·4
99	+ 9·8	+ 2·0	13·42	13·04	11·7	26	+12·9	+ 3·7	13·26	13·37	17·4
218	+ 4·5	+ 4·0	12·61	12·33	11·7	219	+ 4·5	+ 3·1	13·70	13·85	17·9
87	+10·3	+ 5·1	13·93	13·49	11·8	25	+13·1	+ 7·0	13·83	13·96	18·0
130	+ 8·7	+ 4·3	14·12	13·65	11·8	1	+14·3	+ 3·6	13·35	14·17	18·2
194	+ 6·4	+ 5·6	11·84	11·65	11·8	4	+13·9	+ 2·3	13·35	14·20	18·2
126	+ 8·7	− 1·6	12·99	12·69	11·9	37	+12·5	+ 7·0	13·60	14·30	18·3
145	+ 8·1	+ 1·1	12·99	12·69	11·9						

N.G.C. 6645

R.A. = $18^h\ 26^m.8$, Dec. = − 16° 58′ (1900·0)

This cluster is mentioned in the catalogues of Shapley, Trumpler and Collinder, and was investigated by Raab and Trumpler.

The star counting gave 230 as the total number within the region. The distribution of stars within the cluster region and its surroundings results in a background density of 5320 stars per square degree (an indication that the cluster lies in a rich cloud part)

and in 92 cluster stars within a cluster region of 10′·9 diameter. Fig. 1 gives the diagram of the star density per square degree within the cluster region.

The figure gives, of course, only the average distribution. Actually, this cluster does not look very regular, empty regions changing with very dense aggregations. The most remarkable fact is an eccentrically situated condensation composed of the stars:

Star	X	Y	Star	X	Y
146	+8·2	+4·3	158	+7·8	+4·2
148	+8·1	+4·7	161	+7·8	+4·5
152	+8·0	+4·6	163	+7·7	+4·2
153	+8·0	+4·2	165	+7·6	+4·6
155	+7·9	+4·3	166	+7·6	+4·2

where X and Y correspond to the co-ordinates of Table V. This secondary star agglomeration forms a small cluster of globular form with the centre at (+7·9, +4·4) and a diameter of 0′·6 (=0·5 parsecs). It was therefore impossible to measure the image diameters on the photographic plate. The photovisual magnitudes are between $13^{m}\cdot5$ and $14^{m}\cdot2$. (An estimation of photographic magnitudes makes it probable that the CI of these stars coincide with those of the other cluster stars.) The space density is at least equal to the central density of M 11 * (one of the richest among the galactic clusters), determined at a limiting star magnitude of $15^{m}\cdot5$.

Table V gives the distribution of $m - M$. We see again, as with Tr. 33, that there appear two ranges, the first between 177 and 127, the second between 199 and 131. We again suppose the stars included in the first part to be giants and find it supported by Trumpler, who found 9 giants among the observed 36 stars. A rearrangement with changed M (not given here) shows that the 8 stars concerned fit well within the second part as giants.

The mean distance modulus of the cluster group is 11·99, corresponding to

$$r = 1860\ (2510) \pm 130 \text{ parsecs}, \qquad d = 10'\cdot9 = 5\cdot9\ (8) \text{ parsecs}.$$

The results of the previous investigations were:

	Distance	Diameter	
	Parsecs	′	Parsecs
Shapley	1450–2200	10	6·4
Trumpler	1230	13	5·6
Collinder	2380	12 × 10	7·6

The number of cluster stars, after correction for the background stars, is 90 (among them 8 giants) against 92 got statistically. The magnitude distribution is given in the following table:—

m_{pv}	10	11	12	13	> 13·5
Number of stars	3	5	5	22	55

Norman Lockyer Observatory,
Sidmouth:
1943 *January* 7.

* Trumpler, *L.O.B.*, 12, 14, Table 6, 1925.

THE THEORY OF CEPHEID VARIABLES AND NOVÆ

F. Hoyle and R. A. Lyttleton

(Received 1942 September 3)

Summary

A new theory of Cepheid variables is developed from the consideration of a binary system in which a common atmosphere surrounds both components. The major part of the mass of either component is regarded as being confined to a volume small compared with that of the whole system. The material of the atmosphere does not take part in the orbital motion of the components. In the discussion here presented, in sections 1 and 2 a priori evidence quite apart from the agreement of the dependent theory with observation is advanced for the plausibility of this model. In section 3 the amplitude of the oscillation of regions lying in the orbital plane of the binary is discussed and in section 4 this investigation is extended to give theoretical velocity-time curves for the motion of the free surface. General considerations are advanced in section 5 that enable the properties of the whole star to be obtained from the results of sections 3 and 4 which refer only to the equatorial regions. In section 6 the physical properties of the system are considered and it is shown that variations in surface temperature and apparent luminosity will accompany the dynamical oscillations of the atmosphere. The paper then considers the properties of these stars as a group and in section 7 it is shown that they will occupy a peculiar position in the luminosity-type diagram similar to that for observed Cepheids. In sections 8 and 9 the theory leads to explanations of the $P\sqrt{\rho}$ law and the period-luminosity relation. The well-known asymmetry and phase lag of the light curves of the Cepheids are discussed in section 10 and some general remarks on the whole question are made in section 11. Next, in section 12, the subsequent development of binary systems with a common atmosphere is considered and it is shown that a possible explanation of the occurrence of novæ arises naturally as a further stage in the evolution of these stars.

The initial treatment here given does not constitute a complete account of the very intricate problems that arise from the present theory. It attempts however to outlive those features of the proposed mechanism that seem of chief importance and it has been found possible to develop the theory sufficiently for extensive comparison with observation to be made.

---

1. *Binary Systems with a Common Atmosphere.*—In a recent paper an ingenious interpretation of the β Lyræ system has been proposed by Kuiper * and several features of this star are explained in terms of the dynamical properties of a binary system possessing a single atmosphere entirely surrounding both components. The investigation is applied explicitly to β Lyræ though it is evidently recognised that the model could also be used for other individual systems presenting similar difficulties. It would seem however that a binary system with components sharing a common atmosphere is more important even than this and represents an essential stage in the evolution of binary systems.

According to the accretion theory of stellar evolution, binary systems evolve from wide to close pairs as the mass increases and for components of masses M_A and M_B moving in nearly circular orbits, the period P and separation a at any time are given by

$$P \propto (M_A+M_B)/M_A^3M_B^3 \qquad a \propto (M_A+M_B)/M_A^2M_B^2. \tag{1}$$

---

* *Ap., J.*, 93, 133, 1941.

These relations express the fact that in almost all cases the added material contributes negligible angular momentum to the system. With continued increase of mass the separation must eventually diminish until it is comparable with the sum of the radii of the two components and the system would reach the neighbourhood of the contact configurations described in Kuiper's paper, where illustrations are given showing that a stage would occur at which material from one component flows through the atmosphere at and near the neutral point of the critical equipotential surface. If the flow continues a binary system with a single enveloping atmosphere would be bound to form. On this basis the evolution from the stage of distinct separate components to a system surrounded by a common atmosphere becomes a definite accountable process and the existence of such stars is an essential theoretical requirement, that is the theory predicts that these stars must exist.

When both components are main series stars of similar compositions it can be shown that the common atmosphere is supplied entirely by the more massive component. This means that in the evolution of the system a stage is reached at which the more massive component extends beyond the critical point of the figure-of-eight equipotential, while the less massive component remains wholly within its share of this surface. One form of proof of this statement is given in Kuiper's paper and a further independent proof based on the evolutionary point of view can easily be constructed. The important point in these proofs is that for a main series star of given mass and uniform composition the radius is uniquely determined. If however the compositions of the components involve certain differences of internal distribution one or both of them may have radius considerably greater than a main series star of the same mass. In this case the above situation may be altered. For clearly if the less massive component were a giant it might be the first to exceed its share of the critical surface and it would then supply material to the other component. Particular examples of this type of star would seem to be β Persei and u Herculis. In the former star curious irregularities in period have long presented an outstanding difficulty and the process of transference of mass may lead to an explanation of some of the peculiarities of this system.

A recent investigation of the structure of giant stars * has shown that the property of large radius is mainly due to high hydrogen content in the atmosphere of the star as compared with the interior. Accordingly the transfer of material from the giant component of smaller mass is likely to result in it losing its giant character to such a degree that the star remains within the limits of its critical surface, at which stage transfer to the more massive component would cease. When both components are giants the transfer of mass will continue until either the giant character of one of the components is removed or a balance is reached with the ratio of the radii in suitable relation to the ratio of the masses. For the case of components of similar compositions and when material is transferred from the more massive component, we should expect that this balance would be achieved when the masses of the components become equal, since on account of the similar compositions the components will then also have equal radii.

2. *The Effect of Accretion on the Common Atmosphere.*—We now consider how further accretion may be expected to affect the common atmosphere once it has formed. If the material being accreted continues to possess negligible angular momentum about the centre of mass of the system then a general braking action on the material of the atmosphere will be produced. This would evidently be most marked in systems in which the atmosphere is exceptionally tenuous, which would be the case when both components are of the giant type. Thus, unless some cause is operating to supply the surface layers with angular momentum it would be proper to assume that the effect of accretion will be to prevent the rotation of the outermost part of the system. When the components are of equal mass and radius the only cause of this kind would seem to be viscosity, the importance of which has been considered by a number of writers.

* *M.N.*, 102, 218, 1942.

It is well known that any extensive redistribution of angular momentum within a star by this means requires a time in excess of 10^{10} years, and this is long compared with the time required for accretion to slow down the material of the atmosphere; for in such an interval the mass added would be comparable with the mass of the whole star.

If however the components are unequal material will pass from one component to the other, and the resulting flow of material through the atmosphere may delay or prevent the braking of the rotational motion of the surface layers, since it tends to expose the inner faster moving layers. Thus the favourable conditions for the slowing down of the surface layers are first that the components should be of almost equal mass and radius, and second that the atmosphere should be of very low density. The first of these means that no flow of material occurs in the atmosphere, while the second ensures that accretion acts as an efficient braking mechanism.

It may be noted that when these conditions are fulfilled the process discussed by Kuiper, in which material is ejected from the common atmosphere (fig. 4, p. 149), no longer produces a permanent flow in the atmosphere. For the slowing of the motion of the outer layers removes the centrifugal force responsible for the ejection process and consequently the flow of material necessary to supply the process must also cease.

It may be mentioned also that the ejection process proposed by Kuiper provides a very plausible source of heavy elements in interstellar space and possibly also of the material of the dark nebulæ. The presence in space of such elements as calcium, sodium, carbon, etc., has always appeared something of a difficulty so far as the conception that the stars have evolved from primitive hydrogen alone is concerned, and it is therefore of interest that this suggested process is a step towards resolving this difficulty. At the same time however it is obvious that *any* mechanism tending to remove material from a star will contribute to the interstellar heavy elements and it is therefore possible that the observed material is due to a number of different causes. A second possible source presents itself from the discussion given in section 12 of this paper.

3. *The Periodic Gravitational Field.*—We have seen how a binary system may come to possess a surrounding atmosphere in which accretion destroys the rotation of the outer layers and it is proposed now to consider the properties of such an atmosphere.

We shall neglect any eccentricity in the relative orbit of the two components. This step receives justification in more than one way. In the first place the eccentricities of close spectroscopic binaries as derived from observation show a marked tendency to be small and to decrease with decreasing separation, so that since the stars we are concerned with here are regarded as at a more advanced stage still the assumption of negligibly small eccentricity would appear warranted, and especially so in a first treatment. Secondly, however, the component cores are now moving within a common atmosphere and it is generally accepted that the first dissipative effect would be the damping out of any eccentricity originally present in the orbital motion. The problem is similar to the question of the planetary orbits and the damping of their eccentricities.

Let X be a point in the plane of the orbital motion at a fixed distance x from the centre of gravity G of the whole system. The field at X changes in a periodic manner owing to the motion of the component cores. The force may be divided into a component F_ψ towards G and a component at right angles to this, where ψ is the angle between the line GX and the line AB joining the two cores. It seems clear that radial oscillations of an element of the atmosphere will be brought about mainly through variations in F_ψ with ψ and in the following analysis we shall consider the radial oscillations on the simplifying assumption that it is caused by F_ψ and not by the component of force perpendicular to GX.

The investigation given below refers to the case $M_A = M_B$. In the model conceived we should expect that M_A and M_B would be very nearly the same, in accordance with the discussion of the preceding sections. But even if some slight inequality existed it would not introduce any serious differences from the case of exact equality. This can

be seen most simply by quoting results obtained below for the case of exact equality of the masses. Then, F_ψ has equal maximum values at $\psi = 0$, π and equal minimum values at $\psi = \pi/2$ and $3\pi/2$. The ratio $F_0 : F_{\pi/2}$ is about 3 : 1, and this variation in the radial force is found to produce an oscillation with a total amplitude of only about 10–15 per cent. of the radius. This means that a large change in the radial force produces only a small change in the radius of the star. When M_A/M_B is not unity F_ψ still takes maximum values at $\psi = 0$ and $\psi = \pi$, but F_0 differs from F_π. The amount of the difference at a representative point X can easily be calculated and it is found that the ratio $F_0 : F_\pi$ is of the same order as the ratio $M_A : M_B$. Thus a mass ratio, 4 : 5 say, would mean a similar ratio for successive values of the maximum force. Since however a change in F_ψ by 300 per cent. in the case of equal masses is found to produce only about 15 per cent. change in radius, it is evident that a 20 per cent. variation in F_ψ would produce a negligible change in radius. This conclusion can be verified by direct calculation in a manner exactly analogous to the investigation as given below.

In view of these remarks we shall develop the theory on the basis of equal components and we consider radial oscillations occurring under the radial component of force only. The period of the oscillations will clearly be half that of the orbital motion. The value of $F_\psi(x)$ is

$$F_\psi(x) = \frac{x - \cos\psi}{(x^2 - 2x\cos\psi + 1)^{3/2}} + \frac{x + \cos\psi}{(x^2 + 2x\cos\psi + 1)^{3/2}}.$$

The following table gives the values of F for $\psi = 0°$, 30°, 45°, 60° and 90° for various values of x, where the mass of either component is taken as unit of mass and half the separation of the components as unit of distance:—

TABLE I

x	F_0	F_{30}	F_{45}	F_{60}	F_{90}
1·5	4·16	1·37	0·84	0·63	0·51
1·7	2·18	1·05	·70	·51	·44
1·9	1·35	0·81	·58	·46	·38
2·1	0·93	·63	·48	·39	·33
2·3	·68	·51	·40	·34	·29
2·5	·53	·41	·34	·29	·26
2·7	·42	·34	·29	·25	·23

If r is the radial distance from G of a particular element of the atmosphere when $\psi = 0$, we may denote by $T_0(r)$, $\rho_0(r)$ the temperature and density associated with this element at $\psi = 0$. As the components rotate ψ increases uniformly and we may write $y(r)$ as the radial distance of the same material element and $T_\psi(y)$ and $\rho_\psi(y)$ as the corresponding temperature and density (the distance y is evidently a function of ψ). To determine the oscillation of the atmosphere we have to find $y(r)$, $\rho_\psi(y)$ and $T_\psi(y)$ in terms of r, T_0 and ρ_0. Now in any purely radial motion we have

$$r^2\rho_0(r)dr = y^2\rho_\psi(y)dy \tag{2}$$

and this is the mathematical expression of the assumption that the component of force perpendicular to GX can be neglected for the purpose of discussing the amplitude of the radial oscillations. The solution of the problem is determined by equation (2) together with the hydrostatic equations

$$\left.\begin{aligned} \Re\mu^{-1}\frac{d}{dr}\{\rho_0(r)T_0(r)\} &= -\rho_0(r)F_0(r) \\ \Re\mu^{-1}\frac{d}{dy}\{\rho_\psi(y)T_\psi(y)\} &= -\rho_\psi(y)F_\psi(y) \end{aligned}\right\} \tag{3}$$

where $\Re$ is the gas constant and μ is the mean molecular weight in the atmosphere.

The adoption of equations (3) is justified provided the velocity of pulsation is small compared with the velocity that a particle would acquire in falling freely through a distance comparable with the amplitude of the pulsation. This condition is satisfied both for the solutions of (2) and (3) found below and the observed oscillatory velocities of the Cepheids. Thus the oscillations of the atmosphere are *forced* by the periodic gravitational field. This state of things is completely different from Eddington's pulsation theory in which *free* oscillations of the atmosphere are considered. This is expressed mathematically by the fact that equations (3) are first order equations, whereas the equations of the pulsation theory are second order. The use of first order equations permits of approximation to a far greater extent than would be possible with a second order equation.

Since the Cepheids are stars of much greater dimensions than main series stars of similar luminosities, the value of μ in the atmosphere must lie near 0·5 (except possibly very close to the surface). This means that for regions at temperatures higher than say 10^5 degrees, the ratio of specific heats γ is effectively 5/3 provided radiation pressure does not play more than a small part in the support of the atmosphere. Since the oscillations may be considered as adiabatic except very near the surface, we may write for temperatures greater than 10^5 and for all values of ψ

$$p \propto \rho T \propto \rho^{\gamma}, \tag{4}$$

so that (3) gives

$$\frac{F_0(r)}{F_\psi(y)} = \left\{\frac{\rho_\psi(y)}{\rho_0(r)}\right\}^{2-\gamma} \cdot \frac{d\rho_0(r)}{dr} \Big/ \frac{d\rho_\psi(y)}{dy}. \tag{5}$$

Now from considerations of the structure of the atmosphere of simple stars, for small steps δr and δy, the quantities defined by

$$\delta\rho = \rho(r+\delta r) - \rho(r), \text{ etc.},$$

must satisfy the inequalities

$$-\frac{\delta\rho_0(r)}{\rho_0(r)} > > \frac{\delta r}{r} \quad \text{and} \quad -\frac{\delta\rho_\psi(y)}{\rho_\psi(y)} > > \frac{\delta y}{y}.$$

Provided $\dfrac{dy(r)}{dr}$ is of order unity, as it turns out to be, it follows immediately that $\delta\left(\dfrac{dy}{dr}\right)\Big/\dfrac{dy}{dr}$ is of order $\dfrac{\delta r}{r}$, so that we have from (2)

$$r^2\frac{d\rho_0(r)}{dr} = y^2\frac{d\rho_\psi(y)}{dy} \cdot \left(\frac{dy}{dr}\right)^2,$$

and using this in (5) leads to

$$\frac{F_0(r)}{F_\psi(y)} = \frac{y^2}{r^2} \cdot \left\{\frac{\rho_\psi(y)}{\rho_0(r)}\right\}^{2-\gamma} \cdot \left(\frac{dy}{dr}\right)^2.$$

Finally using (2) and putting $\gamma = 5/3$ this leads to

$$\frac{dy(r)}{dr} = \frac{r^{4/5}F_0^{3/5}(r)}{y^{4/5}F_\psi^{3/5}(y)}. \tag{6}$$

If now any pair of associated values of r and y, r_0 and $y(r_0)$ say, are given, then (6) can be solved to give the complete required solution, since $F_0(r)$ and $F_\psi(y)$ are known functions of r and y. It is essential that such a pair of values be specified since the general solution of (6) for a given value of ψ is a one-parameter family of curves. For the case $\psi = \pi/2$ a number of solutions have been obtained numerically and are given in the following table:—

TABLE II

Solution No.	(1)	(2)	(3)	(4)	(5)	(6)
r	y	y	y	y	y	y
1·7				1·00	1·56	2·14
1·8		0·60	0·80	1·30	1·80	2·41
2·0		1·00	1·30	1·75	2·23	2·82
2·2		1·41	1·68	2·12	2·61	
2·4	0·67	1·74	2·02	2·47	2·96	
2·6	1·06	2·04	2·34	2·76		
2·8	1·38	2·34	2·62			
3·0	1·66	2·62	2·88			
Value of r for which $y=2·0$	3·20	2·57	2·39	2·14	1·90	1·66
$1-\frac{y(r)}{r}$ at $y=2·0$	·36	·23	·16	·07	−·05	−·17

This table brings out the important result that for values of y near to 2·0 the solutions (2) to (6) all lead to amplitudes of oscillation of the same order as the values derived from observations of Cepheids. This shows that in the present theory the variable gravitational field not only provides an understood physical cause for the oscillations, but that it is also capable of giving amplitudes of the required order of magnitude over a considerable range of the solutions satisfying equation (6).

It might be supposed that by extending the analysis to all values of r and using the condition $y(0)=0$, a unique solution could be found. But a difficulty arises, for near $r=1·0$ the approximations used above break down owing to the presence of the massive cores of the binary. The solution and even the definition of the problem in this region involves great complexity, but in the absence of such a solution no theoretical specification of the required pair of values r_0 and $y(r_0)$ seems possible. This recognised weakness in the present theory has not been overcome and it is only by the introduction at this stage of some special assumption that further progress has been made. The justification for any such step, apart of course from any immediate plausibility it may have, must lie in the eventual agreement later reached between the theory and observation. In accordance with these considerations we have adopted for the remainder of the work solution (5) of Table II and this leads to agreement with standard examples of Cepheid variables. It is perhaps significant that it is possible to select a suitable solution without difficulty in this somewhat arbitrary way.

Having chosen a particular solution it remains to decide what value of r is to be associated with the free surface at $\psi=0$. To do this, if it is imagined that the separation of two distinct equal stars is continually diminished, then from what has been said in section 1 it is clear that a common atmosphere will not form until the critical equipotential is reached by their surfaces. At this stage the radius from G to the surface in the direction AB in the selected units is $r=1·8$ units. We shall therefore take $r_s=2·0$ units for the radius to the surface in this direction, corresponding to stars that have *not* advanced much beyond the contact stage. (Since for large values of y the function $F_\psi^{3/5}y^{4/5}$ varies little with changing ψ, the amplitude of oscillation would not be substantial when the distance between the components is small compared with the general size of the atmosphere.) With $r_s=2·0$ the solution (5) then gives $y(r_s)=2·23$, so that the motion has an amplitude of about 10 per cent. of the mean radius for elements of the atmosphere lying in the plane of the orbital motion.

Since the details of the solution in any actual case must depend upon the particular properties of the atmosphere it is clear that some variation in the amplitude must be expected for different stars. Thus the foregoing solution can only be regarded as an average case and in individual instances the appropriate solution might deviate somewhat from it.

4. *The Velocity Curves of Cepheids.*—From solution (5) it is seen that when $\psi=90°$ the material at $y=1·8$ units is unaltered in radial distance from its position for $\psi=0°$.

For values greater than this the material moves outwards between $\psi = 0$ and $\psi = 90°$ on the whole, while for smaller values it moves inwards in this interval. To find the radius for intermediate values of ψ it is again necessary to specify a pair of values of r and y and for this purpose it would seem permissible to suppose that material at central distance 1·8 units remains unaltered in position throughout the cycle. By means of this assumption it is then possible to find $y(r_s)$ for any value of ψ in the same way as already used for the $\psi = 90°$ case. The following table shows the solutions so found for a number of values:—

TABLE III

r	30° $y(r)$	45° $y(r)$	60° $y(r)$	90° $y(r)$
1·80	1·800	1·800	1·800	1·800
1·85	1·875	1·895	1·909	1·915
1·90	1·946	1·980	2·009	2·025
1·95	2·013	2·065	2·103	2·130
2·00	2·078	2·147	2·190	2·230
2·05	2·142	2·225	2·276	2·327
2·10	2·205	2·300	2·360	2·418

Since $r_s = 2·0$ we have the following values for the extension of the surface for the values of ψ shown:—

ψ	0°	30°	45°	60°	90°
$y(r_s)$	2·000	2·078	2·147	2·191	2·230

In this way with the help of similar calculations for other values of ψ a curve is obtained for the relation between the radius and ψ. The gradient of this curve as a function of ψ gives a quantity directly proportional to the velocity of motion of the surface since the angle ψ increases uniformly with time. The actual velocity v is given by

$$v(\psi) = \tfrac{1}{2}a\frac{d\psi}{dt} \cdot \frac{dy(r_s)}{d\psi} = \frac{\pi a}{2p} \cdot \frac{dy(r_s)}{d\psi} \qquad (7)$$

where a is the separation of the masses and $2p$ is the orbital period. Having found $dy(r_s)/d\psi$ the quantity $pv(\psi)/a$ is obtained as a function of ψ. The results are given in the following table:—

TABLE IV

ψ	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
pv/a	0	·15	·33	·45	·38	·30	·21	·14	·07	0

Since minimum radius occurs at $\psi = 0$, it is seen that $v(\psi)$ as given by this table shows the characteristic form of the observed velocity curves of the Cepheid stars. In addition when p and a are known the numerical value of $v(\psi)$ can be obtained at once. In Table VII below estimates of p and a are given for a number of typical Cepheids and these lead to numerical values of $v(\psi)$ in general agreement with observed values.

5. *The Shape of the Star.*—At the poles of the star the radius must remain unaltered since the radial field does not vary on the axis of the orbital motion. To estimate this radius let Ω_p denote the potential at the surface there. Since the potential at the equator varies from 1·33 at $\psi = 0°$ to 0·82 at $\psi = 90°$ we must have

$$1·33 > \Omega_p > 0·82.$$

For if $\Omega \geqslant 1·33$ there would be a permanent flow from equator to pole of the surface material without any reversal occurring, while if $\Omega_p \leqslant 0·82$ there would be a steady flow from pole to equator. For a suitable intermediate value of Ω_p a stable state may presumably be achieved in which the atmosphere executes an oscillatory surface movement

in period p. As a numerical estimate of Ω_p we take the round number 1·0, which gives polar radius 1·73 units. A precise value for Ω_p is unnecessary since for any reasonably intermediate value we obtain the result that the polar axis is substantially less than 2 units, so that it must always be the least radius of the star.

To a first approximation the surface as a whole may therefore be regarded as having three unequal perpendicular axes, two of these r_1 and r_2 in the orbital plane corresponding to $\psi = 0°$ and $\psi = 90°$ respectively and the third r_3 the polar radius. The free surface may be represented by an ellipsoid of semi-axes

$$r_1 = 2{\cdot}00, \qquad r_2 = 2{\cdot}23, \qquad r_3 = 1{\cdot}73.$$

A mean radius of the star may be defined by $(r_1 r_2 r_3)^{1/3}$ which has the value 2·0 units about and is therefore closely similar to the distance between the centres of the component masses.

The equatorial axes rotate with the orbital motion of the binary, but this occurs by radial movement of the surface and not by a tangential or rotational mass motion. The oscillations though radial are not symmetrical, maximum and minimum extension occurring simultaneously at points 90° apart in the plane of the orbital motion.

6. *Variation in the Emission and the Surface Temperature.*—Since the atmosphere is in motion it is to be expected that there will occur at any point of it a cycle of changes in which heat energy is withdrawn to reappear as potential energy and is then set free again. The kinetic energy of the motion by which this is achieved has been seen to be small and will here be neglected. We proceed to consider the variation of surface temperature at a point in the equatorial plane.

Write $E_0(r)$ and $E_\psi(y)$ for the outward fluxes of energy per unit area at 0 and ψ respectively. If $k_0(r)$ and $k_\psi(y)$ are the coefficients of opacity at r and y, then by the equation of radiative equilibrium we have

$$\frac{E_\psi(y)}{E_0(r)} = \frac{k_0(r)\rho_0(r)}{k_\psi(y)\rho_\psi(y)} \cdot \frac{T_\psi^3(y)}{T_0^3(r)} \cdot \frac{dT_\psi(y)}{dy} \div \frac{dT_0(r)}{dr}. \tag{8}$$

The adiabatic approximation gives

$$\frac{dT_\psi(y)}{dy} \div \frac{dT_0(r)}{dr} = \left(\frac{\rho_\psi(y)}{\rho_0(r)}\right)^{\gamma-2} \frac{d\rho_\psi(y)}{dy} \div \frac{d\rho_0(r)}{dr}$$

and with the methods of reduction used in deriving (6) we readily find

$$\frac{E_\psi(y)}{E_0(r)} = \frac{k_0(r)}{k_\psi(y)} \cdot \left(\frac{r^2}{y^2}\right)^{4\gamma-5} \cdot \left(\frac{dy}{dr}\right)^{4-4}. \tag{9}$$

This equation shows that the ratio E_ψ/E_0 depends on γ rather sensitively. It is known that γ must decrease as the surface is approached until in a critical layer near the surface its value falls to less than 4/3. Thus the ratio E_ψ/E_0 has a value near the surface different from its value in the interior. This does not imply that there is any corresponding change in the mean flux per unit solid angle, which is given by

$$\pi^{-1}x^2 \int_0^\pi E_\psi(x)\, d\psi$$

and must be independent of x, where x is any given fixed distance from G.

If the adiabatic approximation were satisfied at the photosphere the ratio $E(y_s)/E_0(r_s)$ could be obtained by using the photospheric value of γ in (9). The heat content of the photospheric layers is so small however that the adiabatic approximation cannot be satisfied in these regions. The adiabatic approximation will become increasingly valid for temperatures that exceed a certain T_0 determined by the condition that the energy content (including ionisation energy) of all material at temperature less than T_0 is equal

to the total energy emitted by a star during a half period of the atmospheric oscillations. The considerations of section 10 show that the value of T_0 is probably less than 2.10^4 degrees. Therefore the value of $E_\psi(y_s)/E_0(r_s)$ will be given by (9) when a value of γ corresponding to a temperature somewhat less than 2.10^4 is used. According to Unsold * the value of γ in the Sun would be about 1·4 and in the following discussion we have determined the ratio (9) by using this particular value of γ. However, results dependent on the value adopted for γ can give only a general indication of the variation of $E_\psi(y)$ with time. On this basis we then have

$$\frac{E_\psi(y_s)}{E_0(r_s)} = \frac{k_0(r)}{k_\psi(y)} \cdot \left(\frac{r_s}{y_s}\right)^{1\cdot2} \cdot \left(\frac{dy}{dr}\right)_s^{-1\cdot6}$$

where r has value near r_s. When the form of the opacity coefficient is known this ratio is immediately calculable. We consider two cases.

(*a*) Opacity by Kramers' law so that $k \propto \rho T^{-3\cdot5}$.
Then from (2) we have

$$\frac{k_0(r)}{k_\psi(y)} = \left(\frac{r_s}{y_s}\right)^{0\cdot8}\left(\frac{dy}{dr}\right)_s^{-0\cdot4}$$

which reduces the required ratio to

$$\frac{E_\psi(y_s)}{E_0(r_s)} = \left(\frac{r_s}{y_s}\right)^{2}\left(\frac{dy}{dr}\right)_s^{-2}. \qquad (10)$$

The temperature change is therefore given by

$$\frac{T_\psi(y_s)}{T_0(r_s)} = \left(\frac{r_s}{y_s}\right)^{\frac{1}{2}}\left(\frac{dy}{dr}\right)_s^{-\frac{1}{2}}.$$

Since the variation of $y(r)$ with r is known the value of dy/dr is also obtainable. From the present result the following values have been calculated for the case $r_s = 2\cdot0$:—

TABLE V

ψ	30°	45°	60°	90°
$y(r_s)$	2·078	2·147	2·190	2·230
$\left(\frac{dy}{dr}\right)_s$	1·3	1·6	1·8	2·0
$(T_\psi/T_0)_s$	0·86	0·77	0·72	0·67

(*b*) Opacity constant, k independent of ρ and T.
In this case we have at once

$$\frac{T_\psi(y_s)}{T_0(r_s)} = \left(\frac{r_s}{y_s}\right)^{0\cdot3}\left(\frac{dy}{dr}\right)_s^{-0\cdot4}$$

and this result leads to the following values:—

TABLE VI

ψ	30°	45°	60°	90°
T_ψ/T_0	0·90	0·83	0·77	0·73

From these two tables it appears that T_ψ/T_0 is not very sensitive to the form of the opacity coefficient. For $\gamma > 1\cdot4$ the ratio would be reduced below these values while for $\gamma < 1\cdot4$ the ratio would be increased.

This change of temperature refers to points in the equatorial plane. At the poles

* *Physik der Sternatmosphären*, p. 382, 1938.

the surface temperature will be expected to remain constant since there is no periodic field acting there. An estimate of the temperature at the poles can be made by supposing that the emission there of energy per unit solid angle is equal to the average emission per unit solid angle at the equator, which would be the same as assuming that the average emission is the same in all directions. Writing E_P for the energy emitted per unit area at the poles we have

$$r_3^2 E_P = 3E_P = \frac{2}{\pi}\int_0^{\pi/2} y_s^2 E_\psi(y_s)\,d\psi.$$

If the opacity is given by Kramers' law this becomes using (10)

$$E_P = 0{\cdot}84 E_0(r_s)\int_0^{\pi/2}\left(\frac{dy}{dr}\right)_s^{-2} d\psi.$$

This integral can be evaluated numerically from the results of earlier sections. It is found that

$$E_P = 0{\cdot}58 E_0(r_s)$$

and hence

$$T_P = 0{\cdot}87 T_0(r_s).$$

Accordingly the surface temperature at the ends of the three axes may be taken as approximately

$$T_0(r_s) \text{ at } r_1, \qquad 0{\cdot}67 T_0(r_s) \text{ at } r_2, \qquad 0{\cdot}87 T_0(r_s) \text{ at } r_3.$$

Thus if $T_0 = 6000°$ for example, we should have temperatures of about 6000°, 4000° and 5200° at these three parts of the surface.

Before making any comparison of this result with observation it is to be remembered that the line of sight to the Earth may have any direction relative to the orbital plane of the system. Thus if it happened to lie in latitude 30° from this, then when the plane $r_1 r_3$ contains the Earth the observed spectrum would be largely contributed by an emitting surface whose temperature ranges between 5200° and 6000°, and the spectrum would appear composite resembling a blend between about F8 and G2. Similarly when the Earth lies in the $r_2 r_3$ plane the observed spectrum would appear as a blend ranging from about G2 to K0.

The variation in apparent luminosity is also explained, for according to Tables V and VI the surface emission per unit solid angle at the equator is four times as great near r_1 as near r_2 and this must lead to a greater observed emission when the Earth lies in the plane $r_1 r_3$ than when it lies in the plane $r_2 r_3$. The order of magnitude of the difference may be expected to be of about one magnitude. The actual amount of the change must depend on the relative direction of the Earth and would be greatest when the latter is in the plane of the orbital motion, and zero when it is in the polar direction. It will be noticed of course that there occurs no real variation in the total luminosity; all that happens is that there is more energy emitted in a given direction at one time than another.

Since the amplitude of the oscillation is greatest at the equator and zero at the poles the apparent velocity of the surface will in general follow the same rule as the variation in luminosity, the two quantities decreasing together with increasing latitude. This conclusion shows that the observed variation in luminosity and the velocity of the motion should decrease together with increasing latitude of the Earth's direction (from star to star) and thus on the present theory we are led to expect that Cepheids should show a strong correlation between these factors. Such a correlation has been discovered by Joy * in his comprehensive survey of these stars.

* *Ap. J.*, **86**, 363, 1937.

7. *The $P\sqrt{\rho}$ Law.*—If the separation of the components is denoted by a then Kepler's law applied to the orbital motion gives

$$4\pi^2 a^3 = 2GM(2p)^2$$

where G is the constant of gravitation and M the mass of either component, while $2p$ is the orbital period so that p is the observed period of the oscillations. If R is the average radius of the system, which in appropriate units corresponds closely with $(r_1 r_2 r_3)^{1/3}$ as has been seen, the average density ρ is given by

$$\tfrac{4}{3}\pi\rho R^3 = 2M.$$

It is clear that M can be eliminated from these two equations to give

$$p^2\rho = 3{\cdot}5 \times 10^7 (a/R)^3$$

when c.g.s. units are used. If P denotes the period of the Cepheid in days the relation becomes

$$P\sqrt{\rho} = 0{\cdot}07(a/R)^{3/2}.$$

In section 3 it has been seen that R is approximately equal to a, so that for the present model

$$P\sqrt{\rho} = 0{\cdot}07.$$

Evidently some variation in this quantity may arise from star to star through slight differences of the ratio a/R from unity.

8. *The Position of the Cepheids in the Luminosity-Type Diagram.*—Regarding the two components as equal, the configuration that either would adopt if alone as an individual star may be denoted by $(L, r;\ T_e)$, any two of these quantities determining the third by means of the proportionality

$$r^2 T_e^{\,4} \propto L. \qquad (11)$$

It has been seen that the average radius of the combined star is about $2r$, so that its configuration may be denoted by $(2L, 2r;\ T_e')$ and it therefore follows that the average effective temperature is given by

$$T_e' = 0{\cdot}84\,T_e.$$

Thus whereas either component existing alone would appear at the point (L, T_e) of the diagram, the combined system will appear at $(2L, 0{\cdot}84\,T_e)$. This represents a shift of considerable extent in a direction roughly transverse to the line of the main series and taking the point higher in the diagram and towards the right in the usual form of the figure. Moreover this displacement would occur whether the original components were giants or main series stars and therefore it would be expected that the Cepheids as a group will occupy a peculiar position in the luminosity-type diagram.

9. *The Relation between Period and Luminosity.*—The numerical value of the separation has been seen to correspond closely to the average radius of the star and this can be estimated from observation by means of (11) by using the effective temperature corresponding to the mean spectral type and with L put equal to the average luminosity. Values of R obtained in this way are given by Russell, Dugan and Stewart, *Astronomy*, p. 768, and also by Eddington, *Internal Constitution of the Stars*, p. 182, but these estimates make no allowance for space absorption which reduces the measured luminosities below their true values. To make some adjustment for this we have increased the luminosities by 0·5 magnitudes above their usual values and this has the effect of increasing the radius by about 25 per cent.

In this table the third column is calculated as explained above. The fourth column shows the total mass $2M$. These values are found by applying Kepler's equation to the quantities given in the second and third columns, it being remembered that the

TABLE VII

	Period(o) d	Separation(c)	$2M(c)$	$L(c)$	$L(o)$	$11 \times 10^{17} apL^{-1}$
RR Lyra	0·57	$5·6 \times 10^{11}$	5·4	−1·9	−0·9	·04
SU Cass	1·95	$1·2 \times 10^{12}$	4·5	−1·1	−1·7	·15
α UMin	3·97	$2·1 \times 10^{12}$	5·8	−2·2	−2·3	·30
δ Ceph	5·37	$2·8 \times 10^{12}$	7·6	−3·2	−2·8	·34
η Aql	7·18	$3·5 \times 10^{12}$	8·3	−3·5	−3·2	·39
ζ Gemi	10·15	$4·2 \times 10^{12}$	7·2	−3·0	−3·9	·34
X Cygn	16·38	$6·3 \times 10^{12}$	9·3	−3·9	−4·5	·48
Y Ophi	17·12	$7·0 \times 10^{12}$	11·6	−4·7	−4·7	·47
l Cari	35·52	$1·3 \times 10^{13}$	17·3	−6·2	−6·0	·54

orbital period is twice the observed period of Cepheid oscillations. This procedure automatically fixes $P\sqrt{\rho}$ at 0·07, whereas in practice it may differ slightly from 0·07 on account of the separation not being exactly equal to R. The luminosity can then be calculated directly using the methods described in a previous paper.* The value 1·4 has been adopted for the mean molecular weight μ in constructing column five in the above table. It will be seen that the differences between the calculated and observed luminosities are no more than would be likely to arise through μ being somewhat different from 1·4, or through the separation of the components being somewhat different from the values given in Table VII. It is possibly of some significance that the agreement between the observed and calculated luminosities requires a fairly high value of μ, for according to a recent analysis of giant stars this is a favourable condition for their development.† It may also be noted that giant stars in general tend to be less luminous than main series stars of the same mass and internal composition. Accordingly the calculated luminosities will be somewhat too high, since the methods used apply primarily to main series stars. The value 1·4 may therefore be expected to underestimate μ in the central parts of the components.

10. *The Asymmetry of the Light Curves.*—Originating with Unsöld's work a number of advances have recently been made in the study of the surface layers of stars. In particular the application to Cepheid variables has been considered by Eddington ‡ on the basis of the pulsation theory. The investigation given below rests on similar concepts but the method adopted and the results arrived at are of a different nature.

The most striking feature of the light curves of the majority of Cepheids is the rapid rise to maximum compared with the slow fall. This characteristic property can be explained on the present theory as a consequence of the variation with ψ of the amount of neutral hydrogen in the atmosphere. In order to calculate the required quantity in any element of solid angle $d\omega$ it is necessary for the relation between temperature and pressure in the outer layers to be known. Thus we need the values of the total pressure P and the electron pressure P_e as functions of T_ψ for each value of ψ and for $T_\psi \leqslant 20{,}000°$. This requirement will be met by supposing that P and P_e have the same dependence on T as that determined by Unsöld for the Sun.§ For this assumption to be valid even as an approximation certain conditions, discussed below, have to be satisfied. The values adopted are in Table VIII.

Here P and P_e are measured in dynes per sq. cm. The last column Y gives the fraction of hydrogen in neutral form and has been derived by means of Saha's equation using the values given in the first and third columns. This table of values will be assumed to hold for all ψ. It is seen that for $T < 8000°$ all the hydrogen may be regarded as in

* *M.N.*, 102, 177, 1942.
† *M.N.*, 102, 218, 1942.
‡ *M.N.*, 101, 182, 1941.
§ *Physik der Sternatmosphären*, p. 144, Table 36 (1938).

TABLE VIII

$T°$	$\log_{10} P$	$\log_{10} P_e$	Y
7,000	3·91	2·37	1·00
8,000	4·07	2·72	0·99
9,000	4·18	3·16	·95
10,000	4·27	3·43	·86
11,000	4·33	3·71	·72
12,000	4·38	3·95	·48
13,000	4·43	4·04	·25
14,000	4·47	4·06	·14
15,000	4·51	4·07	·08
16,000	4·56	4·08	·04
17,000	4·60	4·09	0·01

neutral form. The mass of neutral hydrogen at temperature less than 8000° overlying 1 sq. cm. is therefore given by

$$XP(8000°)g_\psi^{-1} = 1{\cdot}18 \cdot 10^4 Xg_\psi^{-1} \text{ gm.}$$

where X is the mass fraction of hydrogen per gram of stellar material in the surface layers and g_ψ is the value of gravity at the surface, constant through the depth concerned but varying throughout the cycle and accordingly a function of ψ. On the other hand for $T > 17{,}000°$ the hydrogen is effectively wholly ionised so that material at temperature greater than this makes a negligible contribution to the total amount of neutral hydrogen. The contribution of the regions between 8000° and 17,000° is

$$Xg_\psi^{-1}\int_{8,000}^{17,000} Y(T)\frac{dP(T)}{dT}dT \text{ gm. per cm.}^2.$$

To evaluate this integral the curve of $P(T)$ may be drawn and its gradient gives dP/dT. The expression is readily found to be

$$1{\cdot}25 \times 10^4 Xg_\psi^{-1} \text{ gm. per cm.}^2.$$

Hence adding these two amounts we obtain

$$2{\cdot}43 \times 10^4 Xg_\psi^{-1} \cdot a^2\left\{\frac{y(r_s)}{2{\cdot}0}\right\}^2 d\omega$$

for the amount of neutral hydrogen in an element of solid angle $d\omega$, where a is the separation of the components in centimetres. By means of the values already given for $y(r_s)$ we obtain the quantities:

$$\psi = 0° \qquad 2{\cdot}43 \times 10^4 Xa^2 d\omega g_0^{-1} \text{ gm.}$$
$$\psi = 90° \qquad 2{\cdot}43 \times 10^4 X(1{\cdot}25a^2) d\omega g_{90}^{-1} \text{ gm.}$$

There is thus an excess of neutral hydrogen in a given $d\omega$ for $\psi = 90°$ as compared with $\psi = 0°$. Now we also have

$$g_\psi = 4F_\psi(y_s)GM/a^2$$

where M is the mass of either component in gms. Since $F_0(r_s) = 1{\cdot}091$ and $F_{90}(y_s) = {\cdot}306$ the excess is found to be

$$1{\cdot}92 \times 10^4 Xa^4(GM)^{-1}d\omega \text{ gm.}$$

The energy required to ionise this mass of neutral hydrogen is

$$2{\cdot}5 \times 10^{17} Xa^4(GM)^{-1}d\omega \text{ ergs}$$

and it is evident that the energy for this ionisation must be abstracted from the outward flux of radiation through the element $d\omega$ between the times of maximum and minimum

radius, whilst a similar amount must augment the outward flux between minimum and maximum radius. The total energy moving outwards from the interior of the star during a half period is

$$pLd\omega/8\pi \text{ ergs}$$

where L is the average luminosity. The ratio between this outward flux and the energy required to ionise the neutral hydrogen is

$$11 \times 10^{17} Xap/L$$

the term in GM being eliminated by means of Kepler's equation. Numerical values of this ratio, apart from the factor X, are given in the last column of Table VII. The quantity X may be expected to be fairly close to unity since the value involved here refers to the surface layers only, where the concentration of hydrogen is likely to be much greater than at even moderate depths. From the table it is seen that the above ratio has a marked tendency to increase with increasing period, so that the present investigation shows that the degree of asymmetry of the light curves should increase with period.

The present results provide evidence in favour of the use of the adiabatic approximation for material at temperature exceeding 20,000°, since material at temperature less than this has an energy content in the form of ionisation energy that is comparable with the total energy radiated during the period of an oscillation.

We now consider briefly the conditions under which the density and temperature in the surface layers can be related in the manner shown in Table VIII. The structure of these layers is determined by the following equations:—

$$\left.\begin{aligned} \tfrac{4}{3}acT^3\frac{dT}{dr} &= -k\rho H \\ \frac{\Re}{\mu}\cdot\frac{d}{dr}(\rho T) &= -g\rho \\ H &= \tfrac{1}{4}acT_e^{\,4} \end{aligned}\right\} \qquad (12)$$

where T_e is the effective temperature at the surface, a is Stefan's constant and c the velocity of light. The coefficient of opacity k depends on ρ, T together with the parameter X. The value of k can be specified for values of ρ, T and X within wide limits, by using numerical results also given by Unsöld.

We can construct two sets of equations by inserting the appropriate values of g, H and X in (12), first for a typical Cepheid and second for the Sun. It is evident that if the values given in Table VIII are to be applicable to the Cepheid in question these two sets of equations must yield the same dependence of ρ on T. This will be approximately true if the quantity g/kH has roughly the same value in both sets of equations. The value of g for the Sun is $2{\cdot}74 \times 10^4$ while from the values given in Table VII the acceleration g is of order 10^2 for a typical Cepheid. Therefore we require the value of kH in the Cepheid to be less than its value for the Sun by a factor of order 10^2. This reduction must be achieved almost entirely by decreasing k, since H has practically the same value in the two cases. Considering first the material at temperature less than 8000°, Unsöld's numerical values for k show that the necessary decrease can be brought about by increasing X from its value in the Sun to a value near unity. The resulting decrease in k in this instance is due to two causes, for first the opacity of the heavy elements is greater than that of hydrogen, while second the opacity of both heavy elements and hydrogen decreases with decreasing electron pressure. Thus increasing X has the effect of reducing k partly by decreasing the electron pressure and partly by decreasing the proportion of heavy elements. Furthermore to reduce k by a factor 10^{-2} it is only necessary to diminish the concentration of the heavy elements by a factor of about 10^{-1}.

It follows therefore that provided the outer atmospheres of the Cepheids contain only a few per cent. by mass of heavy elements the adoption of Table VIII for $T < 8000°$ is justified.

For $T > 10{,}000°$ a much greater reduction in the concentration of heavy elements is required if Table VIII is to be applicable. This arises because whereas for $T < 8000°$ most of the free electrons are supplied by heavy elements, for $T > 10{,}000°$ the majority of the free electrons come from the ionisation of the hydrogen. Thus for $T > 10{,}000°$ the electron pressure increases with increasing hydrogen content, and hence if the concentration of heavy elements is adjusted to make the values of P given in Table VIII valid for $T < 8000°$ then the values of P given will be too high for $T > 10{,}000°$. Conversely if the concentration of hydrogen is altered to make the values of P applicable for $T > 10{,}000°$, then the values given for $T < 8000°$ will be too low. This situation does not have any serious effect on the investigation given earlier in this section for two reasons. First the values actually used for P must be at any rate a good approximation over the whole range of T, since according to Table VIII the pressure varies by a factor less than 4 over the whole relevant range of T. Also, it has been seen that almost equal contributions are made to the amount of neutral hydrogen in the atmosphere first by material at temperature less than 8000° and second by material at temperature greater than this. Accordingly if Table VIII gives correct values either for $T < 8000°$ or for $T > 8000°$ it follows that the order of magnitude of the amount of neutral hydrogen will be unchanged from the value calculated above.

Two further points of interest may be mentioned. The assumption that Table VIII may be used for all ψ is easily seen to be justified provided the table is applicable for some one value of ψ. This can be shown by considering g/kH as a function of ψ. The variation in g with ψ is almost exactly compensated by the variation of H with ψ, both quantities varying by a factor of about 3 between $\psi = 0°$ and $\psi = 90°$. Therefore g/kH is practically independent of ψ since X and therefore k is independent of ψ.

It may be concluded that provided the concentration of heavy elements is sufficiently small near the surface the asymmetry and phase lag of the light curves of Cepheids may be explained along the lines indicated above. If on the other hand the proportion of heavy elements were too high, then in such cases the light curves would tend to be symmetrical. This dependence of the degree of asymmetry on the particular composition of the surface layers seems to provide a plausible explanation of the ζ Geminorum class of Cepheids. These stars possess the exceptional property that their light curves are approximately symmetrical. Thus the present theory not only explains the standard form of the light curves but also shows how on occasion exceptional cases may arise.

11. At this stage it seems desirable to add a number of remarks of a general nature by way of reviewing the foregoing theory of Cepheid variables.

It is evident that a rigorous discussion of the properties of the stellar systems here considered constitutes a problem of exceptional difficulty. On the other hand the semi-quantitative reasoning alternatively resorted to here appears to furnish ample evidence in favour of the theory as a whole. In the first place the model suggested is not introduced *ad hoc* to provide the mechanism of the changes but is a necessary stage in the theoretical evolutionary scheme of double stars. In the varying gravitational field it immediately introduces a recognisable and understood cause of the oscillations, so that their occurrence is an essential feature for such stars, and it is in no way a mechanism that requires further special assumption to explain why it is inoperative in the atmosphere of stars otherwise similar to Cepheids. According to the theory Cepheids should be of comparable but rarer occurrence than close spectroscopic binaries. The present analysis, while deficient in many respects, shows that the theory is potentially capable of explaining quantitatively such features as the asymmetrical velocity and light curves and also the period-luminosity law.

For the theory to be tenable it is necessary that there should be sufficient room for

the components to pursue their orbits freely within the common atmosphere. This requirement is clearly most likely to be fulfilled for components of giant type, which would mean that they consist of small dense cores surrounded by large tenuous atmospheres. Thus the combined system provides perhaps the most favourable conditions both as regards the room for the motion and bringing damping to a minimum. It may also be remarked that the same process that leads to the addition of hydrogen to the atmosphere so that the stars become giants is also responsible for the decrease in the separation of the components.

There exists a remarkable observational fact associated with Cepheids as a class that lends very powerful support to the present view of the internal binary nature of these stars. Since Cepheids are here regarded as being formed by two very close components of a binary system but surrounded by a common atmosphere so that they do not appear double, it will be expected that such stars will be without any observable close companion. That is, a Cepheid considered as a single stellar system should not, except perhaps in rare instances, be a component of a close binary of distinct components. It is of great significance therefore that of some 200 examples of Cepheids available none is known possessing a close companion, whereas at least one in six of stars of the same spectral type as Cepheids and otherwise selected at random does possess a close companion.

12. *Evolution of a Binary System beyond the Cepheid Stage.*—The effect of further accretion by the system is to reduce the separation of the components and ultimately therefore a fusion of the components into a single body must occur. Owing to the conservation of angular momentum, originally largely orbital, the single body so formed would rotate at a speed comparable with that of the orbital motion just before coalescence begins. Two possibilities then arise. First, if the mass-ratio just before they unite exceeds 3 : 1 (taking the figure known for the case of uniform density) a stable rotating body will be formed. If on the other hand the mass-ratio is less than 3 : 1 the combined body would be rotationally unstable. This second case appears much more likely to occur, for not only does the mass-ratio tend to be near unity in close binary systems but increase of density towards the centres of the components would also seem to favour the second outcome.

In the event of an unstable body being formed, the only way in which stability can be restored is by the ejection to infinity of a portion of the main rotating mass. The mass-ratio of the pieces produced by the break-up must now exceed 3 : 1. The process of ejection is a rapid dynamical motion, not a slow secular development through equilibrium configurations. Regions of the star at very high temperatures, 10^6 or even 10^7 degrees, would almost instantaneously become exposed when the main body of the star is torn asunder, and radiation from surfaces at temperatures of this order will escape. Thus for a time the star will emit radiation at a rate comparable with that of a star of surface temperature higher than 10^6 degrees.

To estimate the visual luminosity likely to occur in such circumstances, let T and T' denote the surface temperatures before and during the outburst, and assume that the emitting surface has approximately the same area throughout. The spectrum of the radiated energy during the outburst is given by the Planck distribution corresponding to an emission temperature T'. In the usual notation this is

$$I(\nu)d\nu = A\nu^3\left\{e^{\frac{h\nu}{kT'}} - 1\right\}^{-1} d\nu.$$

The total energy radiated is $\int_0^\infty I(\nu)d\nu = 6A(kT'/h)^4$ ergs per cm.^{2} per sec. The emission in the visible part of the spectrum is very nearly equal to

$$\int_0^{1.2\times 10^{15}} I(\nu)d\nu \text{ ergs per cm.}^2 \text{ per sec.}$$

and for large values of T' this is effectively

$$A\int_0^{1\cdot2\times10^{15}} \nu^3 \cdot \frac{kT'}{h\nu}d\nu = 5\cdot8\times10^{44}AkT'/h \text{ ergs per cm.}^2 \text{ per sec.}$$

Hence

$$\frac{\text{Visual luminosity of nova}}{\text{Bolometric luminosity of nova}} = 10^{44} \cdot (h/kT')^3.$$

Since the ratio of the bolometric luminosity of the nova to that of the star before the outburst is $(T'/T)^4$ we therefore have

$$\frac{\text{Visual luminosity of nova}}{\text{Bolometric luminosity before outburst}} = 10^{44}\left(\frac{h}{k}\right)^3 \cdot T'/T^4 = 10^{13}T'/T^4. \qquad (13)$$

If in this result we put $T' = 10^7$ degrees, then for $T = 10^4$ degrees the increase in luminosity given by (13) is 10 magnitudes, while for $T = 6000°$ the increase is about 12 magnitudes.

The fact that (13) has only a small dependence on T' may be of importance in explaining why there is so little variation in the observed value of this ratio from one nova to another.

Although the only way in which complete stability can be reached by the system is through the hyperbolic ejection of part of the mass, it does not seem inevitable that the projected fragment will escape the first time it is thrown away from the main mass. On occasion it might not quite reach the necessary speed and it would then describe a closely elliptical path eventually returning to within its initial distance of the main body but now with velocity of approach. The two pieces would temporarily reunite, some of the kinetic energy of motion being degraded into internal heat and lost as extra radiation, but the total angular momentum would be unaltered. This sequence can only result in rendering the single mass more unstable and therefore in time a second break-up must occur of a more violent character than the first. Clearly the process of ejection with subsequent reuniting will cease only when the relative velocity of separation reaches hyperbolic value and a sufficient amount of angular momentum is carried by this motion to allow the remaining mass to be stable. If this tentative suggestion is sound it seems possible that a star might go through the nova phenomenon on more than one occasion within a period of a few years as a result of the same condition of rotational stability.

Accompanying the fission there would almost certainly take place a streaming out between the two main components of a great deal of atmospheric gases. Indeed there would appear to be indirect evidence that small bodies of planetary mass would form by condensation in the material drawn out between the separating masses. Since any such material must possess angular momentum per unit mass about the main body to an amount approaching that possessed by the escaping fragment, at least a portion of this material will form an expanded shell surrounding the primary, while a similar extended atmosphere may come to be associated with the smaller component too. This process would seem to afford a possible explanation of the fairly permanent shells of gaseous material that form the planetary nebulæ. It will be seen also that some of the material drawn out between the separating pieces may escape from them both and the mechanism would thus provide a further source of heavy elements in interstellar space.

OCCULTATIONS OBSERVED AT THE ROYAL OBSERVATORY, CAPE OF GOOD HOPE, IN 1941 AND 1942

(Communicated by H.M. Astronomer)

(Received 1943 April 6)

The occultations were reduced by the method and with the constants given in the *Nautical Almanac*.

Observers:—Dr. J. Jackson J. Mr. A. Menzies. A.M.
Dr. R. H. Stoy. S. Mr. W. P. Hirst. H.
Mr. J. W. Jackson. J.J.

Co-ordinates of instruments used:

Instrument	Longitude	Latitude
	h m s	° ′ ″
10-inch Equatorial	1 13 54·70 E.	−33 56 07·8
18- ,, ,,	54·57	06·8
7- ,, ,,	54·64	07·0
6- ,, ,,	54·69	07·1

For all these instruments we may take

$$\rho \cos \phi' = 0{\cdot}83054 \qquad \rho \sin \phi' = -0{\cdot}55509$$

An analysis of the equations of condition for the two years gave the following corrections to the orbital longitude ΔL and latitude ΔB:—

Mean Epoch	ΔL	Wt.	ΔB	Wt.
	″		″	
1941·41	+0·77	31·7	−1·22	9·8
1942·52	+0·29	25·3	−1·63	10·0

No.	Date 1941	Star	Mag.	Phase	Inst.	Obs.	U.T. h m s
1	Jan. 9	68 Tauri	4·2	D.D.	6-inch	S.	21 21 16·3
2	,, 9	,,	,,	,,	7- ,,	H.	21 21 16·4
3	,, 11	71 Orionis	5·2	,,	10- ,,	S.	22 24 51·8
4	,, 22	γ Libræ	4·0	D.B.	7- ,,	S.	2 04 11·7
5	,, 22	,,	,,	,,	10- ,,	A.M.	2 04 13·1
6	,, 22	,,	,,	R.D.	10- ,,	J.	3 10 52·9
7	,, 22	,,	,,	,,	7- ,,	S.	3 10 53·1
8	,, 23	ϕ Ophiuchi	4·4	,,	10- ,,	S.	1 56 29·8
9	Feb. 13	τ Leonis	5·2	,,	10- ,,	S.	22 51 28·1
10	Mar. 16	B.D. −8° 3689	6·5	,,	10- ,,	S.	3 24 27·9
11	Apr. 3	B.D. −18° 1040	7·1	D.D.	10- ,,	S.	17 51 26·2
12	,, 3	,,	,,	,,	7- ,,	H.	17 51 26·3
13	,, 8	48 Leonis	5·2	,,	7- ,,	S.	21 27 34·9
14	,, 8	,,	,,	,,	10- ,,	J.	21 27 35·3
15	,, 9	83 Leonis	6·5	,,	6- ,,	S.	18 36 53·7
16	,, 9	,,	,,	,,	10- ,,	J.	18 36 53·8
17	,, 9	,,	,,	,,	7- ,,	H.	18 36 53·9
18	,, 9	τ Leonis	5·2	,,	7- ,,	H.	19 23 11·3
19	,, 9	,,	,,	,,	10- ,,	J.	19 23 11·6
20	,, 9	,,	,,	,,	6- ,,	S.	19 23 11·7

No.	Date 1941	Star	Mag.	Phase.	Inst.	Obs.	U.T. h m s
21	Apr. 30	B.D. +18° 920	7·5	D.D.	7-inch	S.	16 35 30·6
22	,, 30	B.D. +18° 923	6·7	,,	10- ,,	J.	16 51 57·9
23	,, 30	,,	,,	,,	7- ,,	S.	16 51 58·0
24	June 2	37 Sextantis	6·3	,,	10- ,,	J.	18 39 21·8
25	,, 2	B.D. +7° 2358	7·0	,,	10- ,,	J.	19 15 53·8
26	,, 2	,,	,,	,,	7- ,,	H.	19 15 54·1
27	,, 7	γ Libræ	4·0	,,	10- ,,	S.	22 26 18·2
28	,, 30	B.D. +4° 2461	6·7	,,	10- ,,	J.	18 47 15·4
29	,, 30	,,	,,	,,	7- ,,	S.	18 47 15·4
30	July 1	13 Virginis	5·9	,,	10- ,,	S.	18 25 40·3
31	,, 1	,,	,,	,,	7- ,,	H.	18 25 40·5
32	,, 3	95 Virginis	5·5	,,	10- ,,	J.	17 20 55·3
33	,, 3	,,	,,	,,	7- ,,	S.	17 20 55·5
34	,, 3	B.D. −9° 3877	7·3	,,	10- ,,	J.	21 07 21·5
35	,, 3	,,	,,	,,	7- ,,	S.	21 07 22·1
36	,, 4	B.D. −12° 4198	,,	,,	10- ,,	J.	18 28 09·7
37	Aug. 28	B.D. −14° 4250	6·9	,,	10- ,,	S.	20 47 32·6
38	Sept. 24	B.D. −12° 4227	7·0	,,	10- ,,	J.	18 22 14·7
39	Oct. 26	B.D. −15° 5541	7·2	,,	10- ,,	J.	22 01 50·2
40	,, 27	B.D. −13° 5813	6·6	,,	10- ,,	S.	22 44 20·0
41	,, 27	,,	,,	,,	7- ,,	J.	22 44 20·4
42	Nov. 27	B.D. −1° 4514	7·0	,,	10- ,,	J.	18 51 14·7
43	,, 29	B.D. +6° 244	6·9	,,	10- ,,	J.	21 18 34·1
44	,, 29	,,	6·9	,,	7- ,,	S.	21 18 34·4
45	,, 29	B.D. +7° 240	6·7	,,	10- ,,	J.	11 01 41·5
46	,, 29	,,	6·7	,,	7- ,,	S.	22 01 41·6

16. A fainter star occulted $1^m\ 25^s.9$ later.
27. Very bad definition.
34–35. Thick cloud passing.
39. Faint.

Equations of Condition, 1941

No.	$''$			
1–2	$+2·7 = +14·3\Delta\alpha$	$+·10\Delta\delta$	$= +1·00\Delta L$	$-·02\Delta B$
3	$+1·2 = +14·1$	$+·13$	$= +·99$	$+·15$
4–5	$+0·2 = +12·7$	$-·48$	$= +·95$	$-·31$
6–7	$-1·4 = -14·3$	$+·15$	$= -1·00$	$-·04$
8	$-1·5 = -13·1$	$+·41$	$= -·96$	$+·29$
9	$+0·1 = -13·2$	$-·47$	$= -·69$	$-·72$
10	$-3·4 = -3·6$	$+·97$	$= -·49$	$+·87$
11–12	$+1·4 = +14·0$	$-·19$	$= +·98$	$-·19$
13–14	$+1·1 = +8·3$	$-·83$	$= +·77$	$-·63$
15–17	$+2·5 = +6·6$	$-·90$	$= +·69$	$-·72$
18–20	$+1·1 = +5·2$	$-·94$	$= +·62$	$-·79$
21	$+1·3 = +13·6$	$+·30$	$= +·96$	$+·27$
22–23	$+0·4 = +13·5$	$+·30$	$= +·96$	$+·28$
24	$+1·4 = +8·3$	$-·83$	$= +·78$	$-·63$
25–26	$+2·1 = +12·8$	$-·51$	$= +·97$	$-·24$
27	$+0·7 = +13·4$	$-·39$	$= +·98$	$-·21$
28–29	$+1·8 = +5·3$	$-·94$	$= +·63$	$-·78$
30–31	$+0·4 = +14·9$	$-·11$	$= +·98$	$+·21$
32–33	$-1·4 = +11·3$	$-·65$	$= +·91$	$-·41$
34–35	$-1·0 = +11·1$	$-·66$	$= +·90$	$-·44$
36	$+0·7 = +14·4$	$+·17$	$= +·92$	$+·39$
37	$+1·8 = +10·6$	$-·68$	$= +·85$	$-·53$
38	$-1·2 = +9·2$	$+·78$	$= +·45$	$+·89$
39	$-1·1 = +10·6$	$+·68$	$= +·83$	$+·56$
40–41	$+1·0 = +14·6$	$-·09$	$= +·95$	$-·30$
42	$0·0 = +12·5$	$+·56$	$= +·96$	$+·27$
43–44	$+0·1 = +14·1$	$+·33$	$= +1·00$	$+·03$
45–46	$+0·6 = +11·0$	$+·68$	$= +·90$	$+·43$

No.	Date 1942	Star	Mag.	Phase	Inst.	Obs.	U.T. h m s
1	Jan. 22	B.D. +4° 175	6·8	D.D.	10-inch	J.	20 39 59·5
2	,, 22	,,	,,	,,	7- ,,	H.	20 40 01·5
3	,, 22	B.D. +4° 176	7·6	,,	7- ,,	H.	20 40 59·3
4	,, 22	,,	,,	,,	10- ,,	J.	20 41 07·3
5	Feb. 26	B.D. +18° 1593	6·8	,,	10- ,,	S.	22 18 17·0
6 *	Mar. 2	B.D. +8° 2429	...	D.	10- ,,	S.	23 36 21·5
7 *	,, 2	,,	...	,,	7- ,,	H.	23 36 21·5
8 *	,, 3	,,	...	R.	10- ,,	S.	0 13 06·6
9 *	,, 3	,,	...	,,	7- ,,	H.	0 13 06·9
10 *	,, 3	B.D. +7° 2384	...	D.	7- ,,	H.	0 56 27·3
11 *	,, 3	,,	...	,,	10- ,,	S.	0 56 28·1
12 *	,, 3	,,	...	R.B.	10- ,,	S.	1 49 02·6
13	,, 11	B.D. −17° 5203	6·0	R.D.	10- ,,	S.	2 09 53·5
14	,, 28	B.D. +13° 2128	6·8	D.D.	10- ,,	S.	22 22 05·8
15	Apr. 28	γ Virginis *m.*	2·9	,,	7- ,,	H.	19 12 18·2
16	,, 28	,,	2·9	,,	10- ,,	J.	19 12 08·8
17	,, 28	,,	2·9	R.B.	10- ,,	J.	20 19 41·3
18	,, 28	B.D. −0° 2603	6·1	D.D.	10- ,,	J.	20 17 06·1
19	,, 28	,,	6·1	,,	7- ,,	H.	20 17 06·1
20	May 8	λ Capricorni	5·4	R.D.	10- ,,	S.	1 04 22·8
21	June 23	B.D. −5° 3762	6·9	D.D.	10- ,,	J.	20 27 49·8
22	,, 23	B.D. −6° 3887	6·6	,,	10- ,,	J.	21 05 44·4
23	July 24	B.D. −15° 4324	7·1	,,	10- ,,	S.	0 43 02·2
24	,, 24	B.D. −17° 4759	7·4	,,	7- ,,	H.	17 55 56·5
25	,, 24	,,	7·4	,,	18- ,,	J.J.	17 55 56·6
26	,, 24	B.D. −17° 4773 *m.*	6·0	,,	10- ,,	J.	19 20 37·6
27	,, 24	,,	6·0	,,	18- ,,	J.J.	19 20 37·7
28	,, 25	B.D. −18° 4896 *m.*	6·4	,,	10- ,,	S.	17 00 52·4
29	,, 25	B.D. −18° 4988	5·2	,,	10- ,,	S.	23 44 59·5
30	,, 26	ρ Sagittarii	4·0	,,	10- ,,	S.	16 34 22·5
31	,, 31	B.D. −5° 5999	6·4	R.D.	10- ,,	S.	0 40 11·6
32	,, 31	B.D. −4° 5896	6·5	,,	10- ,,	S.	2 16 58·3
33	Aug. 19	B.D. −14° 4314	6·2	D.D.	7- ,,	H.	17 15 14·7
34	,, 19	,,	6·2	,,	10- ,,	J.	17 15 14·8
35	,, 19	,,	6·2	,,	6- ,,	S.	17 15 14·8
36	,, 22	B.D. −18° 5155	6·3	,,	10- ,,	J.	17 46 47·0
37	,, 22	,,	6·3	,,	7- ,,	H.	17 46 47·2
38	,, 30	25 Arietis	6·5	R.D.	10- ,,	S.	22 36 54·8
39	Oct. 15	B.D. −18° 4982	5·8	D.D.	10- ,,	J.	19 44 29·0
40	,, 15	,,	5·8	,,	6- ,,	S.	19 44 29·5
41	,, 15	B.D. −18° 4988	5·2	,,	10- ,,	J.	20 17 37·1
42	,, 15	,,	5·2	,,	6- ,,	S.	20 17 37·5
43	Nov. 14	B.D. −15° 5908	6·4	,,	10- ,,	S.	18 53 53·0
44	,, 15	B.D. −11° 5770	7·0	,,	10- ,,	S.	20 57 20·0
45	,, 16	81 Aquarii	6·4	,,	10- ,,	S.	19 20 23·4
46	,, 16	,,	6·4	,,	18- ,,	J.	19 20 23·4
47	,, 16	82 Aquarii	6·5	,,	10- ,,	S.	20 26 49·6

1–4. Very bad definition. Observations 3 and 4 unreliable.
15. Interval between 2 components 13·7 seconds.
16. Interval between 2 components 14·5 seconds.
17. Interval between 2 components 10·5 seconds. Second component also observed by H. 5·8 seconds after mean observed by J.
20. Through thick cloud.
28. Interval between components 5·1 seconds.
41–42. Cloud. Bad definition.
44. Faint. Cloud. Bad definition.

* Total eclipse of the Moon.

Equations of Condition, 1942

No.	$''$				
1–2	$0{\cdot}0 = +12{\cdot}1\Delta\alpha$	$+{\cdot}59\Delta\delta$	$= +{\cdot}95\Delta L$	$+{\cdot}32\Delta B$	
3–4	$+1{\cdot}6 = +12{\cdot}1$	$+{\cdot}59$	$= +{\cdot}95$	$+{\cdot}32$	
5	$-2{\cdot}0 = +12{\cdot}9$	$-{\cdot}42$	$= +{\cdot}94$	$-{\cdot}34$	
13	$-3{\cdot}4 = -\ 8{\cdot}2$	$+{\cdot}81$	$= -{\cdot}58$	$+{\cdot}82$	
14	$+1{\cdot}2 = +14{\cdot}5$	$+{\cdot}13$	$= +{\cdot}93$	$+{\cdot}36$	
15–16	$+1{\cdot}0 = +\ 7{\cdot}5$	$-{\cdot}87$	$= +{\cdot}75$	$-{\cdot}66$	
17	$-0{\cdot}2 = -15{\cdot}0$	$-{\cdot}07$	$= -{\cdot}92$	$-{\cdot}38$	
18–19	$+1{\cdot}8 = +12{\cdot}4$	$-{\cdot}56$	$= +{\cdot}96$	$-{\cdot}27$	
20	$+1{\cdot}4 = -12{\cdot}7$	$+{\cdot}50$	$= -{\cdot}71$	$+{\cdot}70$	
21	$+0{\cdot}6 = +11{\cdot}1$	$-{\cdot}66$	$= +{\cdot}91$	$-{\cdot}41$	
22	$+1{\cdot}8 = +\ 9{\cdot}6$	$-{\cdot}77$	$= +{\cdot}85$	$-{\cdot}53$	
23	$+0{\cdot}2 = +13{\cdot}3$	$-{\cdot}39$	$= +{\cdot}97$	$-{\cdot}23$	
24–25	$+2{\cdot}6 = +\ 5{\cdot}7$	$-{\cdot}92$	$= +{\cdot}49$	$-{\cdot}87$	
26–27	$-0{\cdot}4 = +14{\cdot}3$	$+{\cdot}06$	$= +{\cdot}99$	$+{\cdot}16$	
28	$-0{\cdot}1 = +10{\cdot}0$	$-{\cdot}71$	$= +{\cdot}72$	$-{\cdot}69$	
29	$-0{\cdot}6 = +13{\cdot}2$	$+{\cdot}36$	$= +{\cdot}93$	$+{\cdot}36$	
30	$+0{\cdot}3 = +13{\cdot}6$	$+{\cdot}30$	$= +{\cdot}97$	$+{\cdot}24$	
31	$+1{\cdot}0 = -12{\cdot}2$	$-{\cdot}57$	$= -{\cdot}96$	$-{\cdot}29$	
32	$0{\cdot}0 = -15{\cdot}0$	$+{\cdot}01$	$= -{\cdot}95$	$+{\cdot}32$	
33–35	$-1{\cdot}8 = +12{\cdot}5$	$-{\cdot}51$	$= +{\cdot}95$	$-{\cdot}32$	
36–37	$+2{\cdot}4 = +\ 8{\cdot}3$	$-{\cdot}82$	$= +{\cdot}55$	$-{\cdot}84$	
38	$+0{\cdot}8 = -12{\cdot}5$	$+{\cdot}54$	$= -{\cdot}66$	$+{\cdot}75$	
39–40	$+2{\cdot}8 = +11{\cdot}6$	$-{\cdot}58$	$= +{\cdot}81$	$-{\cdot}58$	
41–42	$-0{\cdot}7 = +12{\cdot}4$	$+{\cdot}48$	$= +{\cdot}88$	$+{\cdot}48$	
43	$+2{\cdot}4 = +13{\cdot}6$	$-{\cdot}35$	$= +{\cdot}84$	$-{\cdot}54$	
44	$+2{\cdot}6 = +10{\cdot}6$	$-{\cdot}70$	$= +{\cdot}50$	$-{\cdot}87$	
45–46	$+2{\cdot}4 = +14{\cdot}4$	$-{\cdot}26$	$= +{\cdot}84$	$-{\cdot}54$	
47	$-0{\cdot}5 = +\ 8{\cdot}6$	$+{\cdot}81$	$= +{\cdot}80$	$+{\cdot}60$	

OCCULTATIONS OBSERVED AT CAPE TOWN AND DURBAN IN 1941 AND 1942

(Communicated by H.M. Astronomer)

(Received 1943 April 6)

The following observations were reduced by the same method as that employed at the Royal Observatory. (See preceding paper.)

Observations were made by Mr. H. E. Krumm with the $4\frac{1}{2}$-inch equatorial of the Astronomical Society of South Africa in Cape Town at Station A. Mr. A. W. J. Cousins observed either with the 8-inch refractor of Durban Observatory, Station B, or with a 3-inch refractor at his home, Station C. All the observations are disappearances at the dark limb.

	A	B	C
	h m s	h m s	h m s
Longitude	1 13 36·8 E.	2 04 01·2 E.	2 03 56·7 E.
Latitude	−33° 56′ 05″·1	−29° 50′ 47″	−29° 51′ 22″
Altitude (metres)	102	79	140
$\rho \cos \phi'$	0·83055	0·86810	0·86802
$\rho \sin \phi'$	−0·55509	−0·49475	−0·49490

Observations by Mr. Krumm.

No.	Star	Mag.	1941	U.T.	p	q	$\Delta\sigma$
				h m s			″
1	68 Tauri	4·2	Jan. 9	21 21 07·1	+1·00	−·02	+2·8
2	104 Tauri	5·0	Apr. 2	18 09 39·3	+ ·68	+·74	+1·1
3	+18° 1040	7·1	3	17 51 18·5	+ ·98	−·20	+1·2
4	48 Leonis	5·2	8	21 27 30·7	+ ·77	−·63	+1·0
5	83 Leonis	6·5	9	18 36 49·7	+ ·69	−·72	+2·4
6	τ Leonis	5·2	9	19 23 08·1	+ ·62	−·79	+0·7
7	+18° 920	7·5	30	16 35 22·4	+ ·96	+·27	+1·4
8	+18° 923	6·7	30	16 51 50·4	+ ·96	+·28	+0·5
9	γ Libræ	4·0	June 8	22 26 09·4	+ ·98	−·21	+0·8
10	+4° 2461	6·7	30	18 47 13·4	+ ·63	−·78	+1·7
11	13 Virginis	5·9	July 1	18 25 31·4	+ ·98	−·21	+0·5
12	−9° 3877	7·3	3	21 07 16·6	+ ·90	−·44	−1·3
13	−12° 4198	7·3	4	18 27 59·7	+ ·92	+·39	+0·4
14	+6° 244	6·9	Nov. 29	21 18 25·4	+1·00	+·03	−0·2
15	+7° 240	6·7	30	22 01 35·6	+ ·90	+·43	+0·3
			1942				
1	+13° 2128	6·8	Mar. 28	22 21 56·6	+ ·93	+·36	+1·4
2	γ Virginis	2·9	Apr. 28	19 11 55·0	+ ·75	−·66	+1·2
3	γ Virginis			12 11·0			
4	−5° 3762	6·9	June 23	20 27 44·7	+ ·91	−·41	−0·2
5	−6° 3887	6·6	23	21 05 38·6	+ ·85	−·53	+1·8
6	−17° 4759	7·4	July 24	17 55 57·0	+ ·49	−·87	+1·8

Of the following observations by Mr. Cousins Nos. 2, 3, 4 were made at Station B, the others at Station C.

No.	Star	Mag.	1942	U.T.	p	q	$\Delta\sigma$
				h m s			″
1	ρ Leonis	3·8	May 23	16 47 33·5	+ ·98	−·22	+1·5
2	−5° 3762	6·9	June 23	20 44 47·0	+1·00	·00	+1·7
3	−6° 3887	6·6	23	21 18 14·0	+ ·98	−·18	+1·4
4	−9° 3975	6·8	24	18 05 49·0	+ ·81	−·59	+1·1
5	−3° 3462	7·1	July 20	16 11 38·0	+ ·70	+·71	+0·1
6	−8° 3737	6·6	21	16 18 49·0	+ ·29	−·96	+1·0
7	−17° 4759	7·4	24	18 01 55·6	+ ·88	−·47	+1·5
8	−18° 4886	6·4	25	16 25 08·5	+ ·99	+·14	+0·9
9	γ Sagittarii	5·3	25	18 47 03·7	+ ·53	−·85	+2·2
10	−14° 4314	6·2	Aug. 19	17 40 00·0	+1·00	+·06	+1·2
11	96 Aquarii	5·7	Oct. 20	23 10 05·0	+1·00	−·06	+0·1

Errata

G. Alter's papers, *M.N.*, **101**, No. 2 and No. 5 and 6:

p. 84, line 20, *for* 4·5 *read* 6.

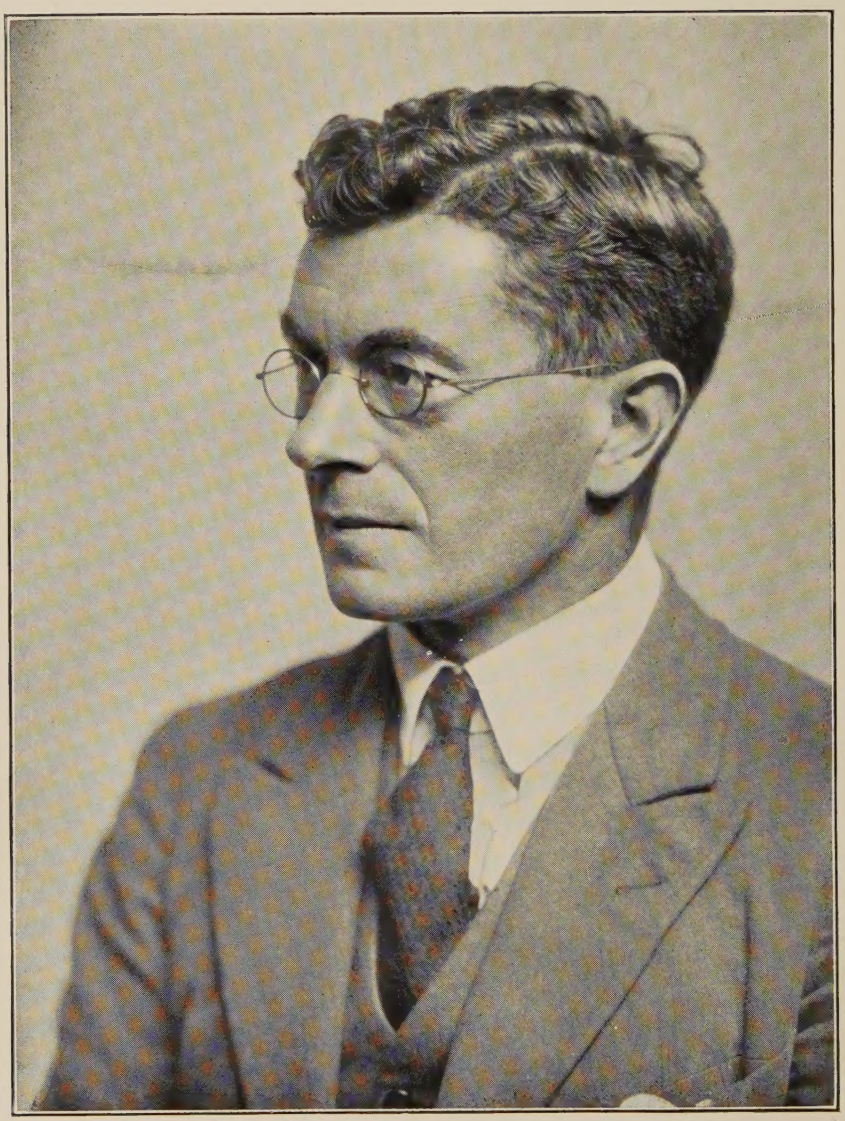
p. 295, last line of Table I, *for* −9·3 *read* +9·3.

M.N., **102**, No. 4:

p. 206, Table II, *for* Parsecs *read* Kpcs.

Cape of Good Hope, Report of the Royal Observatory, *M.N.*, **102**, No. 2:

p. 84, line 8 of § 3, *for* 3″–6″ *read* 3^h–6^h.



Professor S. Chapman, M.A., D.Sc., F.R.S.,
President, 1941–43.